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Generalized Exponential Decay Model for Power–Delay Profiles of Multipath Channels

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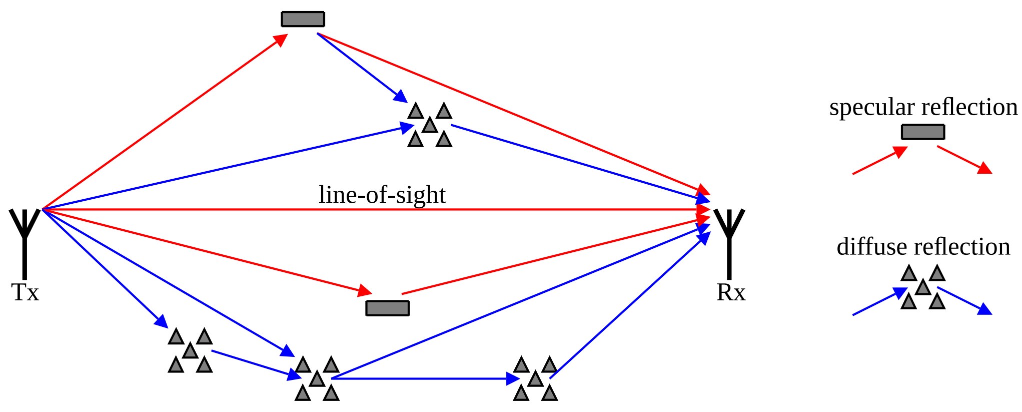
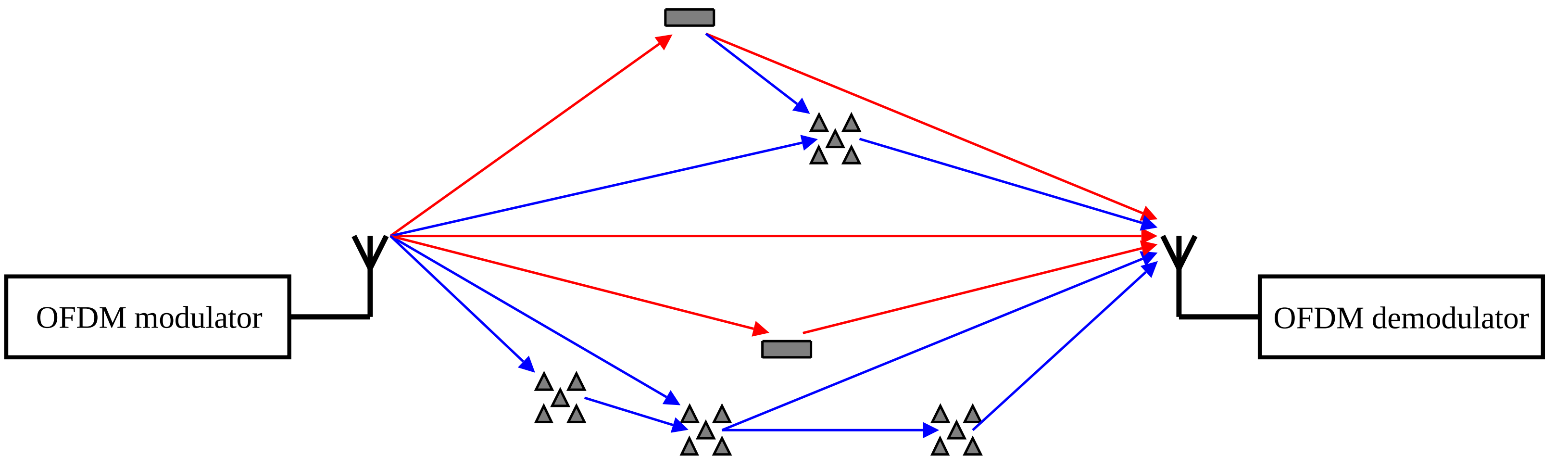
Introduction

Motivation (1)



- Focus: Generic wideband radio channels
  - ▷ Possible line-of-sight path
  - ▷ Specular and diffuse reflections as well as diffraction
  - ▷ Single and multiple scattering
- Assumptions: Quasi-stationary channel with wide-sense stationary uncorrelated scattering (WSSUS)

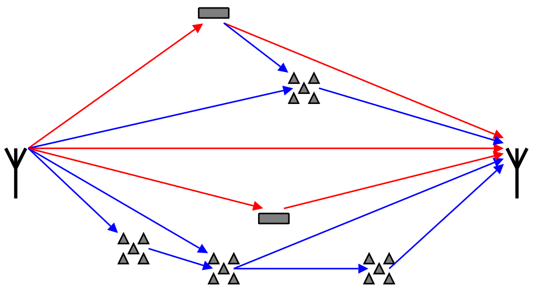
Motivation (2)



- Goals: To develop a generalized model for wideband channels and exploit it for the analysis of OFDM transmission

Power–Delay Profile (PDP)

- Instantaneous vs. statistical channel models
  - ▷ Impulse response: $h(t)$
  - ▷ Power–delay profile: $\boxed{P(t) = \mathcal{E}\{|h(t)|^2\}}$



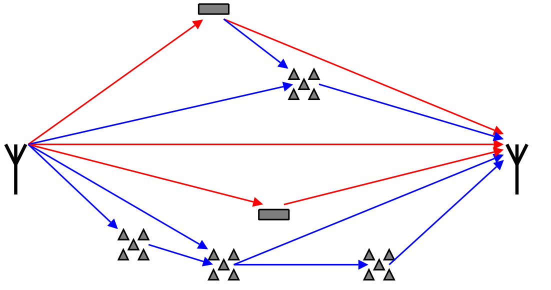
- Characterization of propagation environment
  - ▷ Lag: τ (the largest τ for which $P(t) = 0$ if $t < \tau$)
  - ▷ Channel gain: $g = P_{\mathsf{Rx}}/P_{\mathsf{Tx}} = (1/P_{\mathsf{Tx}}) \int_{-\infty}^{\infty} P(t)\ \mathrm{d}t$
  - ▷ Mean delay: $\mu = (1/P_{\mathsf{Rx}}) \int_{-\infty}^{\infty} tP(t)\ \mathrm{d}t$
  - ▷ Mean square delay spread: $\sigma^2 = (1/P_{\mathsf{Rx}}) \int_{-\infty}^{\infty} (t-\mu)^2 P(t)\ \mathrm{d}t$
  - ▷ Frequency correlation function (FCF), coherence bandwidth
  - ▷ Signal-to-interference ratio (SIR) in OFDM transmission

Continuous Clustered PDPs

Generalized Clustered PDP

- A composite of N clusters:

$$P(t) = P_{\text{specular}}(t) + P_{\text{diffuse}}(t) = \sum_{n=1}^{N} P_{K_n}(t, g_n, \tau_n, S_n)$$

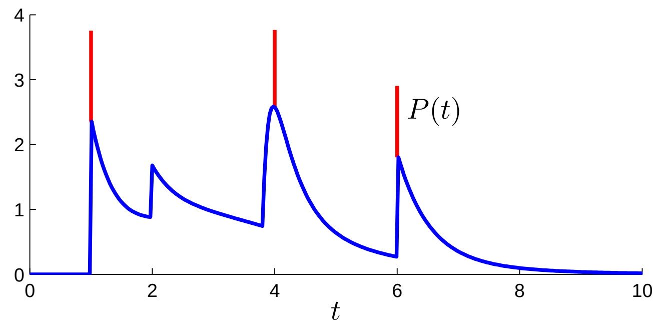


in which for the nth cluster:

  - Order: K_n
  - Gain: g_n
  - Lag: τ_n
  - Delay spread parameters: $S_n = \{\sigma_{n,1}, \sigma_{n,2}, \ldots, \sigma_{n,K_n}\}$

- The total channel gain and lag become $g = \sum_{n=1}^{N} g_n$ and $\tau = \min_n \tau_n$

Example PDP



n	K_n	g_n	τ_n	S_n
1	0	1.4	1.0	$\{\}$
2	0	1.2	4.0	$\{\}$
3	0	1.1	6.0	$\{\}$
4	1	1.2	1.0	$\{0.5\}$
5	1	0.4	2.0	$\{0.5\}$
6	1	0.8	6.0	$\{0.5\}$
7	2	1.2	3.8	$\{0.1, 0.4\}$
8	3	3.0	1.0	$\{0.9, 1.0, 1.1\}$

- Illustrative example with $N = 8$ clusters

$$P(t) = \sum_{n=1}^{N} P_{K_n}(t, g_n, \tau_n, S_n)$$

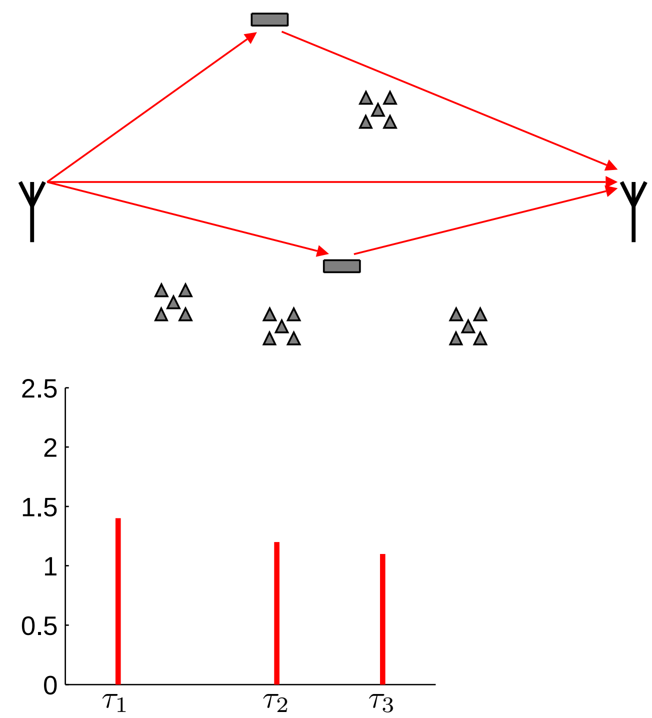
  - ▷ Classic models available for clusters with $K_n = 0$ or $K_n = 1$
  - ▷ New hypoexponential model for clusters with $K_n \geq 2$

Specular Clusters

- $P_{\text{specular}}(t)$ models the potential line-of-sight path and unscattered, mirror-like, reflections ($K_n = 0$)
- Each cluster in $P_{\text{specular}}(t)$ is an impulse:

$$\boxed{P_0(t, g, \tau, \{\}) = g P_{\text{Tx}}\, \delta(t - \tau)}$$

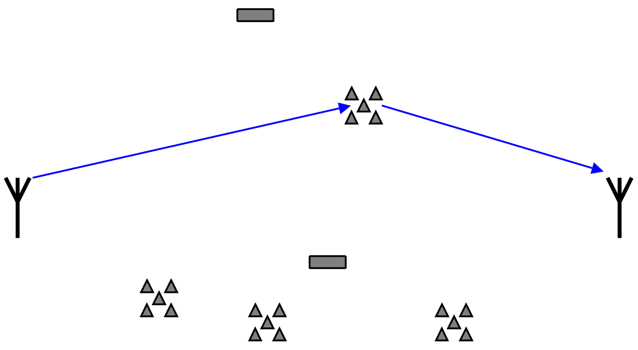
  - Gain: g
  - Lag: τ
  - Mean delay: $\mu = \tau$



Single-Scattering Clusters

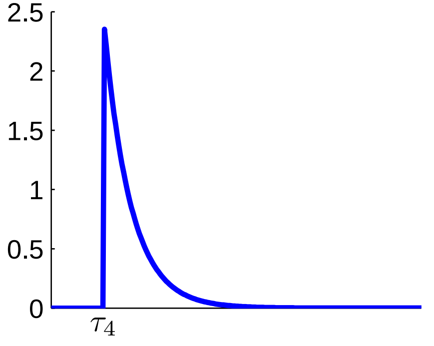
- $P_{\text{diffuse}}(t)$ models the dense multipath components resulting from the combination of single and multiple scattering

- Single scattering ($K_n = 1$) results in the classic (single-)exponential PDP:



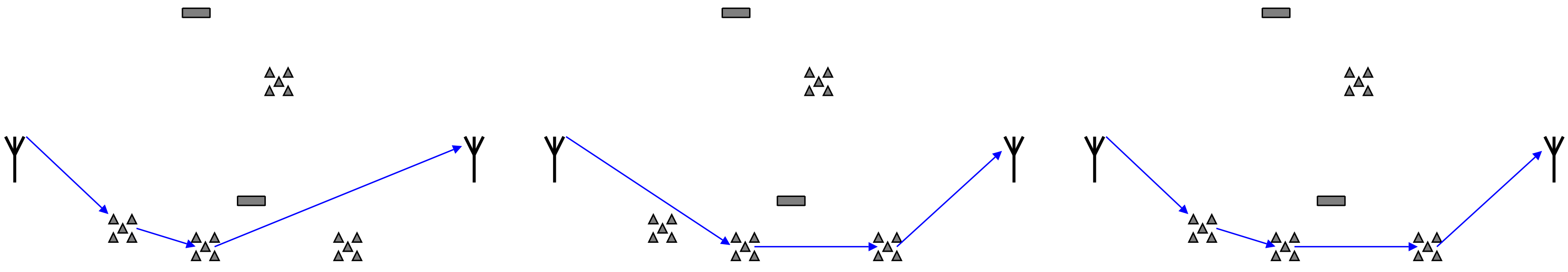
$$P_1(t, g, \tau, \{\sigma\}) = \frac{g P_{\text{Tx}}}{\sigma} e^{-\frac{t-\tau}{\sigma}} \mathrm{U}(t-\tau)$$

  - Gain: g
  - Lag: τ
  - RMS delay spread: σ
  - Mean delay: $\mu = \sigma + \tau$



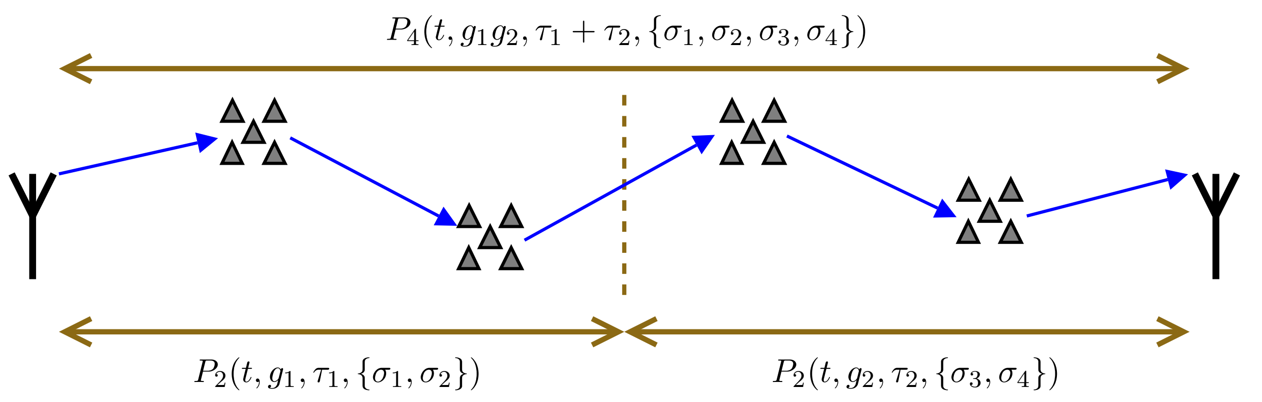
Generalized Exponential PDPs

Multiple-Scattering Clusters



- How to model multiple-scattering clusters ($K_n \geq 2$)?
- Could we represent all clusters (any K_n) in terms of classic single-exponential PDPs ($K_n = 1$)?

PDP Partitioning (1)

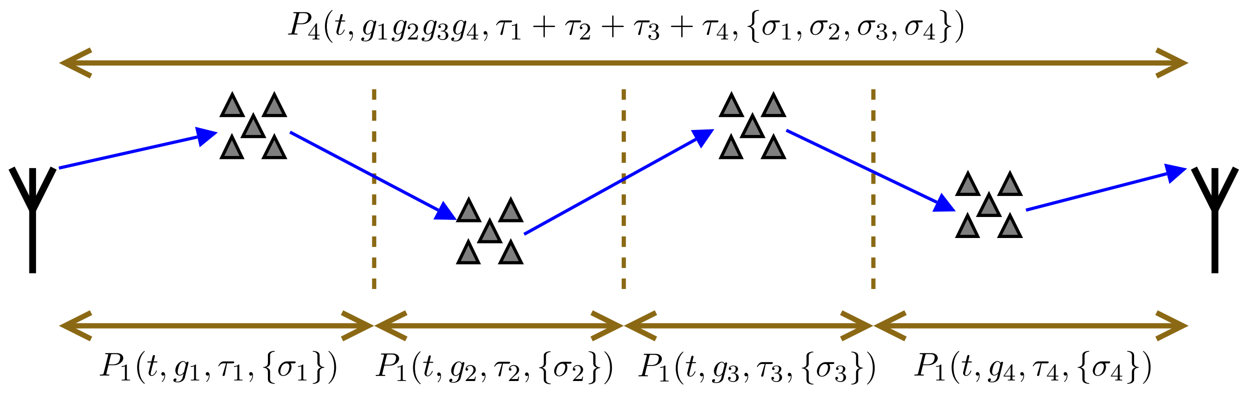


- A cluster with $K_1 + K_2$ reflections can be partitioned into a cascade of two clusters with K_1 and K_2 reflections:

$$\boxed{P_{K_1+K_2}(t, g_1 g_2, \tau_1 + \tau_2, S_1 \cup S_2) = \frac{1}{P_{\mathsf{Tx}}} P_{K_1}(t, g_1, \tau_1, S_1) * P_{K_2}(t, g_2, \tau_2, S_2)}$$

with any $K_1 \geq 0$ and $K_2 \geq 0$

PDP Partitioning (2)

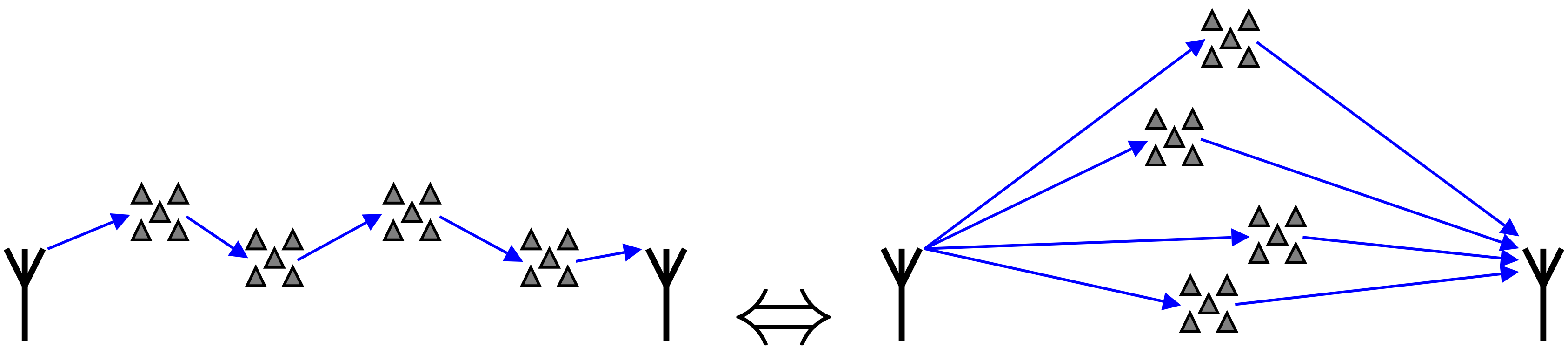


- By applying partitioning repeatedly, a cluster with K reflections becomes a cascade of K single-scattering clusters:

$$P_K(t, g, \tau, S) = \frac{1}{P_{\mathrm{Tx}}^{K-1}} P_1(t, g_1, \tau_1, \{\sigma_1\}) * P_1(t, g_2, \tau_2, \{\sigma_2\}) * \ldots * P_1(t, g_K, \tau_K, \{\sigma_K\})$$

where $g = \prod_{k=1}^{K} g_k$, $\tau = \sum_{k=1}^{K} \tau_k$, and $S = \{\sigma_1, \sigma_2, \ldots, \sigma_K\}$

Hypoexponential PDP

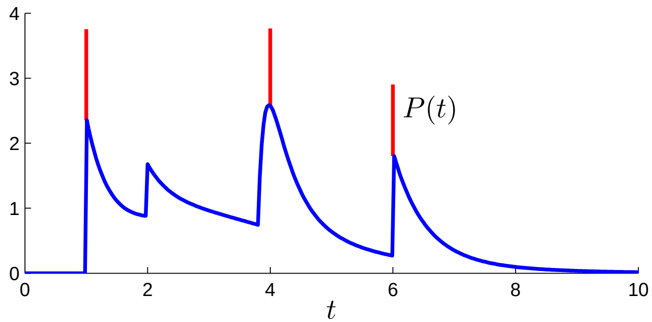


- After calculating the multiple convolutions in the previous slide

$$P_K(t, g, \tau, \{\sigma_1, \sigma_2, \ldots, \sigma_K\}) = \sum_{k=1}^{K} P_1(t, g \times \prod_{\substack{l=1 \\ l \neq k}}^{K} \frac{\sigma_k}{\sigma_k - \sigma_l}, \tau, \{\sigma_k\})$$

  - ▷ K-fold multiple-scattering cluster is effectively equal to the superposition of K single-scattering clusters
  - ▷ Both constructive and destructive interference (positive vs. negative cluster gains)

Updated Example PDP



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7	2	1.2	3.8	$\{0.1, 0.4\}$
8	3	3.0	1.0	$\{0.9, 1.0, 1.1\}$

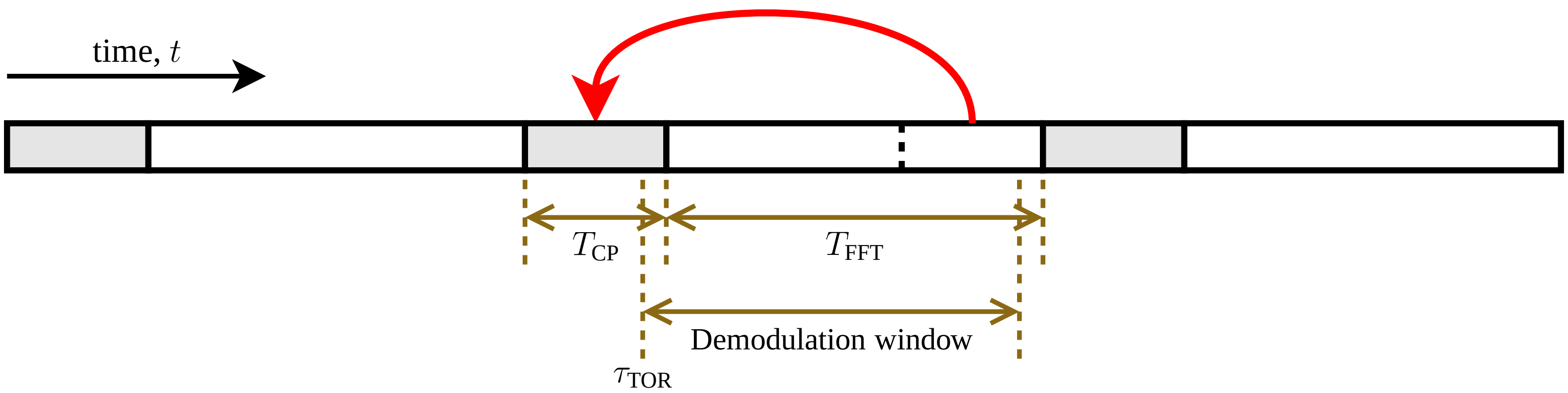
$\Leftrightarrow$

n	K_n	g_n	τ_n	S_n
1	1	1.4	1.0	$\{\epsilon \approx 0\}$
2	1	1.2	4.0	$\{\epsilon \approx 0\}$
3	1	1.1	6.0	$\{\epsilon \approx 0\}$
4	1	1.2	1.0	$\{0.5\}$
5	1	0.4	2.0	$\{0.5\}$
6	1	0.8	6.0	$\{0.5\}$
$7a$	1	-0.4	3.8	$\{0.1\}$
$7b$	1	1.6	3.8	$\{0.4\}$
$8a$	1	121.5	1.0	$\{0.9\}$
$8b$	1	-300.0	1.0	$\{1.0\}$
$8c$	1	181.5	1.0	$\{1.1\}$

- Any generalized exponential PDP can be represented as a sum of single-exponential PDPs ($K_n = 1$)
  - Multiple-scattering clusters ($K_n \geq 2$) based on the hypoexponential PDP
  - Also specular clusters ($K_n = 0$) because $\lim_{\epsilon \to 0} P_1(t, g, \tau, \{\epsilon\}) = P_0(t, g, \tau, \{\})$

Application to Analysis of OFDM Transmission

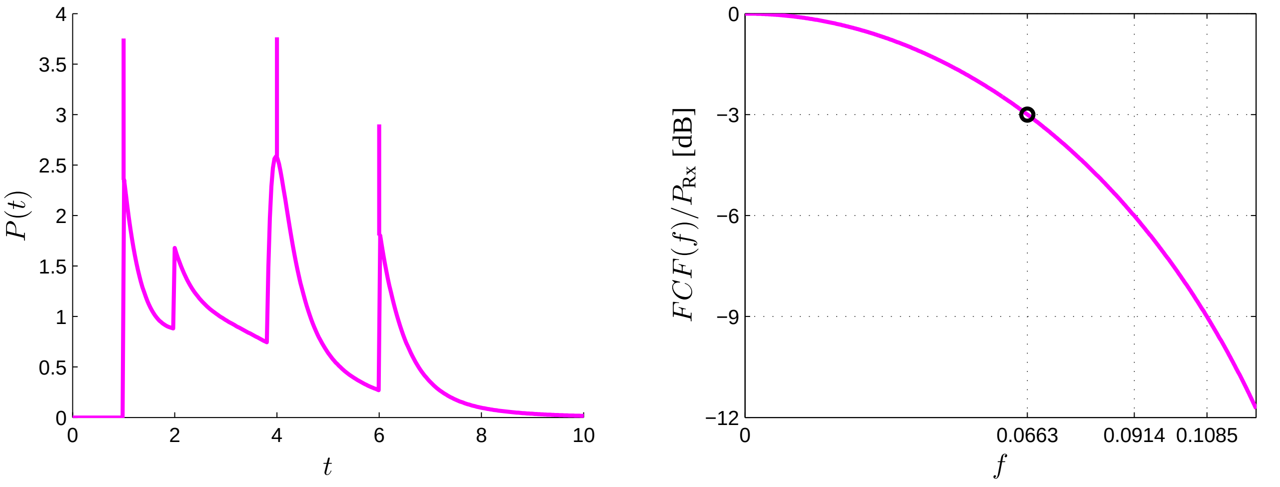
Standard OFDM With Cyclic Prefix



- The usage of fast Fourier transform (FFT) and a cyclic prefix (CP) facilitates simple frequency-domain equalization and robustness against multipath fading
  - Duration of the FFT window: T_{FFT}
  - Duration of the CP window: T_{CP}
  - Symbol demodulation begins at time instant τ_{TOR} which is the time-of-reference (TOR)

Design of OFDM Frequency-Domain Parameters

- Fourier transform of PDP $\Leftrightarrow$ frequency correlation function (FCF):

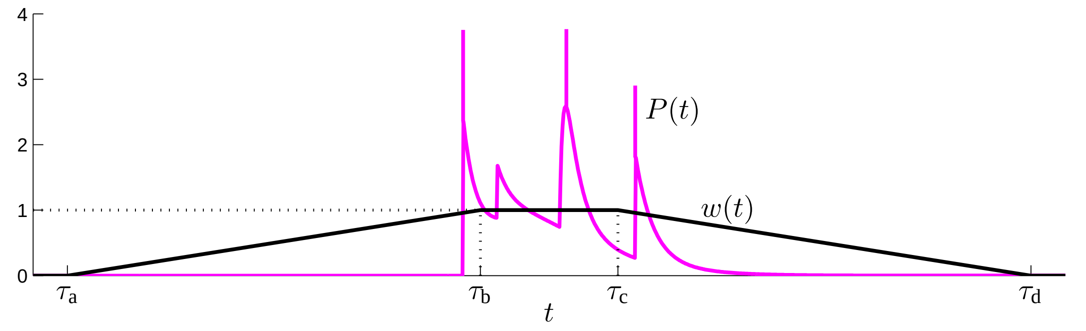


  - ▷ Typically 3dB coherence bandwidth is of particular interest

- Useful information for OFDM system design when choosing subcarrier spacing and pilot structures
  - ▷ Sufficient correlation of adjacent subcarriers/pilots is needed

Signal-to-Interference Ratio in OFDM (1)

- Multipath components that arrive outside of the CP window cause inter-carrier and inter-symbol interference (ICI and ISI)
  - ▷ A windowing approach:



$$w(t) = \frac{1}{T_{\mathrm{FFT}}} \max\left\{0, \min\left\{T_{\mathrm{FFT}}, t - \tau_{\mathrm{a}}, \tau_{\mathrm{d}} - t\right\}\right\}$$

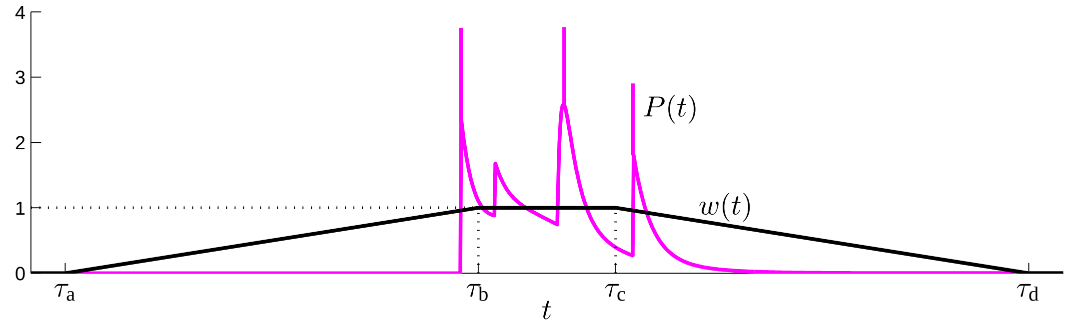
where

$$\begin{cases} \tau_{\mathrm{a}} = \tau_{\mathrm{TOR}} - T_{\mathrm{FFT}} \\ \tau_{\mathrm{b}} = \tau_{\mathrm{TOR}} \\ \tau_{\mathrm{c}} = \tau_{\mathrm{TOR}} + T_{\mathrm{CP}} \\ \tau_{\mathrm{d}} = \tau_{\mathrm{TOR}} + T_{\mathrm{CP}} + T_{\mathrm{FFT}} \end{cases}$$

Signal-to-Interference Ratio in OFDM (2)

- Signal-to-interference ratio:

$$\text{SIR} = \frac{P_{\text{U}}}{P_{\text{ICI}} + P_{\text{ISI}}}$$



where

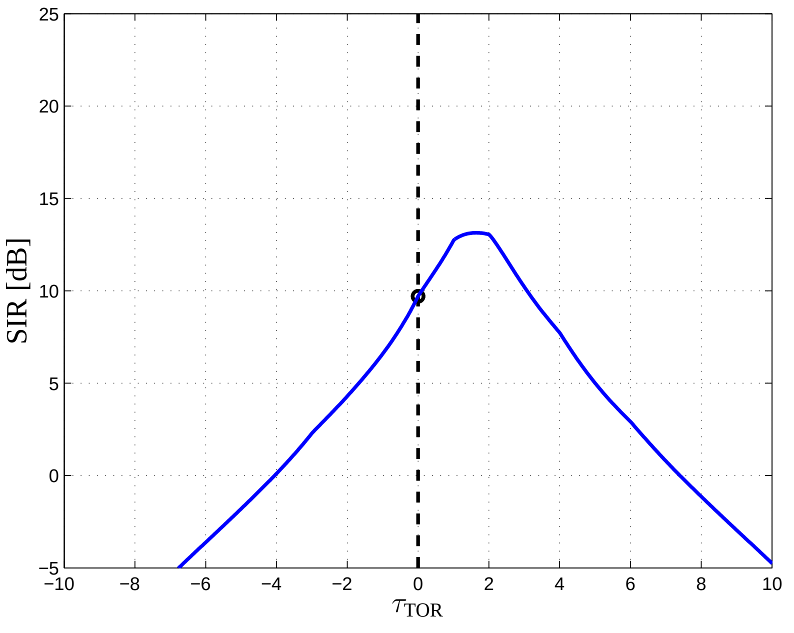
$$P_{\text{U}} = \int_{-\infty}^{\infty} w^2(t) P(t)\ \mathrm{d}t$$

$$P_{\text{ICI}} = \int_{-\infty}^{\infty} w(t) P(t)\ \mathrm{d}t - P_{\text{U}}$$

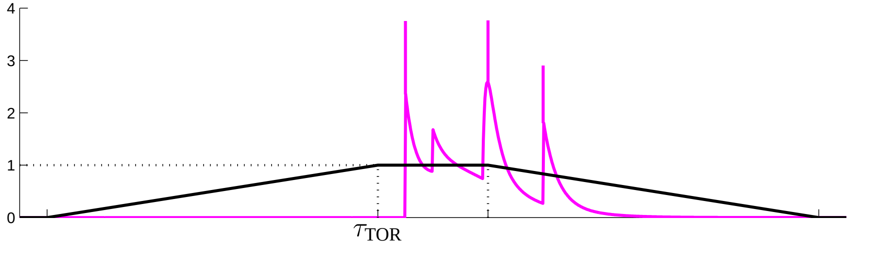
$$P_{\text{ISI}} = \int_{-\infty}^{\infty} \big(1 - w(t)\big)\ P(t)\ \mathrm{d}t$$

- Neat closed-form solutions available for these integrals!

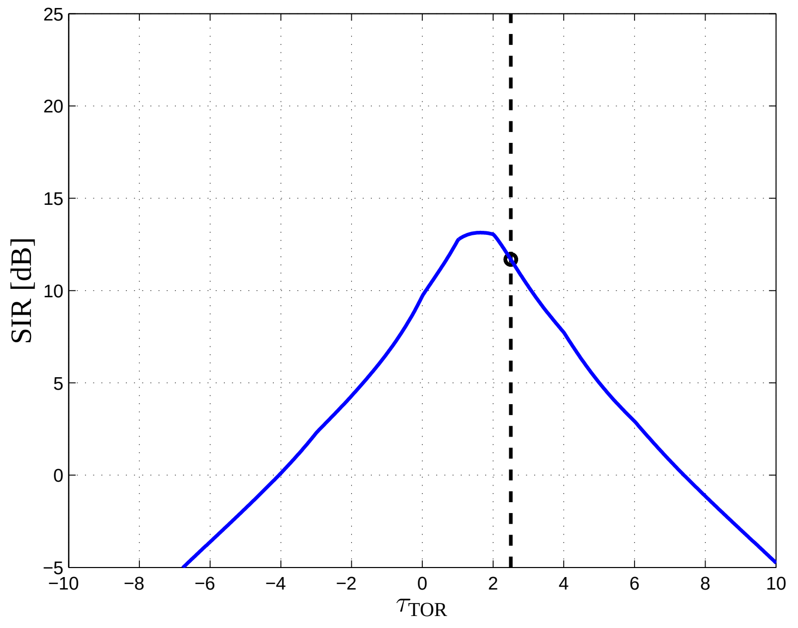
Time Synchronization in OFDM (1)



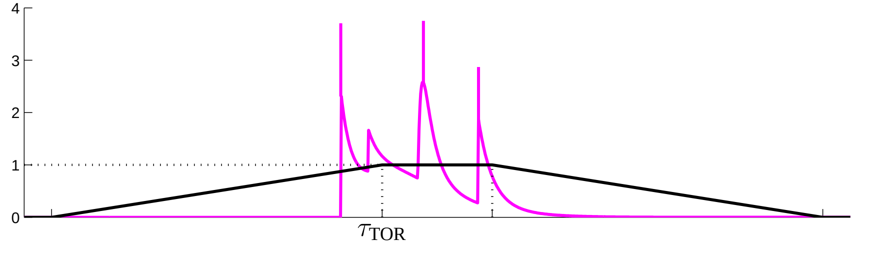
- Too early time-of-reference ($\tau_{\text{TOR}} = 0.0$):



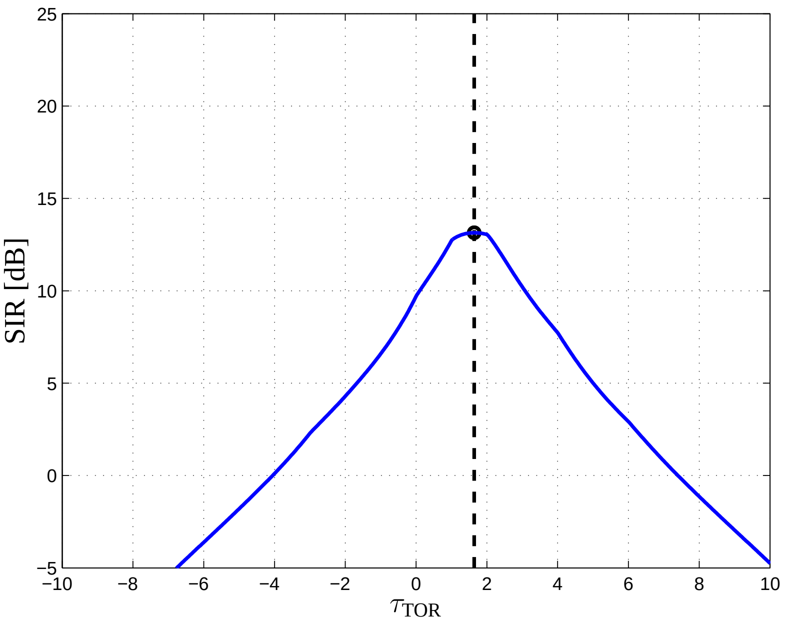
Time Synchronization in OFDM (2)



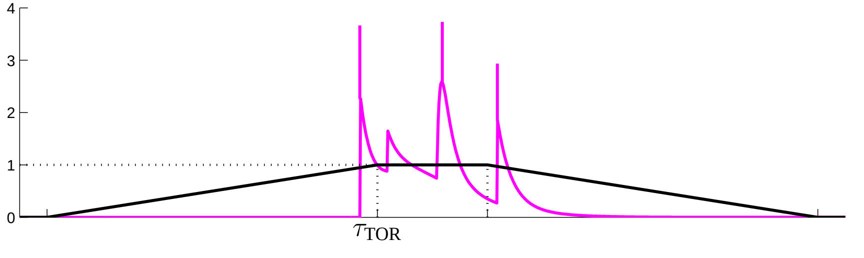
- Too late time-of-reference ($\tau_{\mathrm{TOR}} = 2.5$):



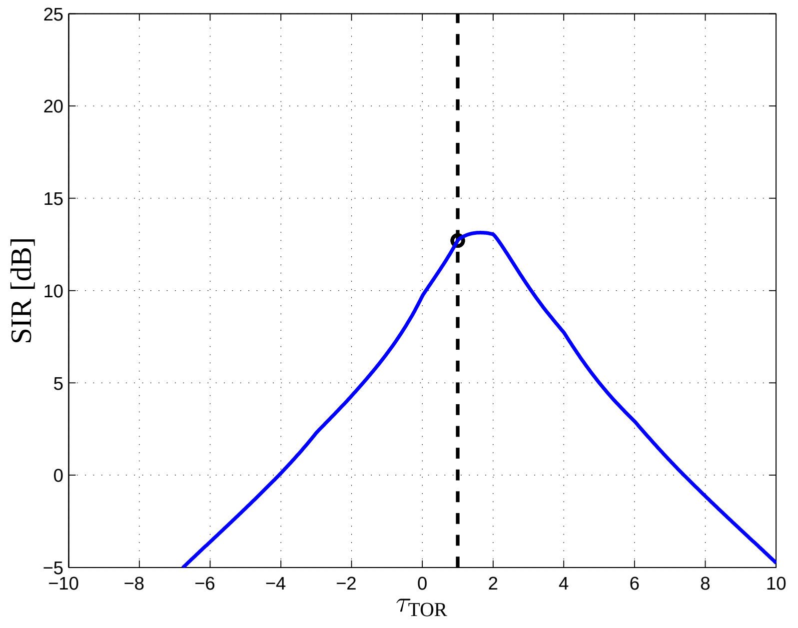
Time Synchronization in OFDM (3)



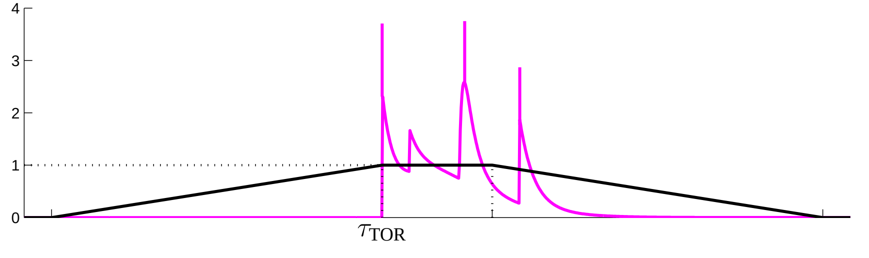
- Optimal time-of-reference ($\tau_{\text{TOR}} = 1.64$):



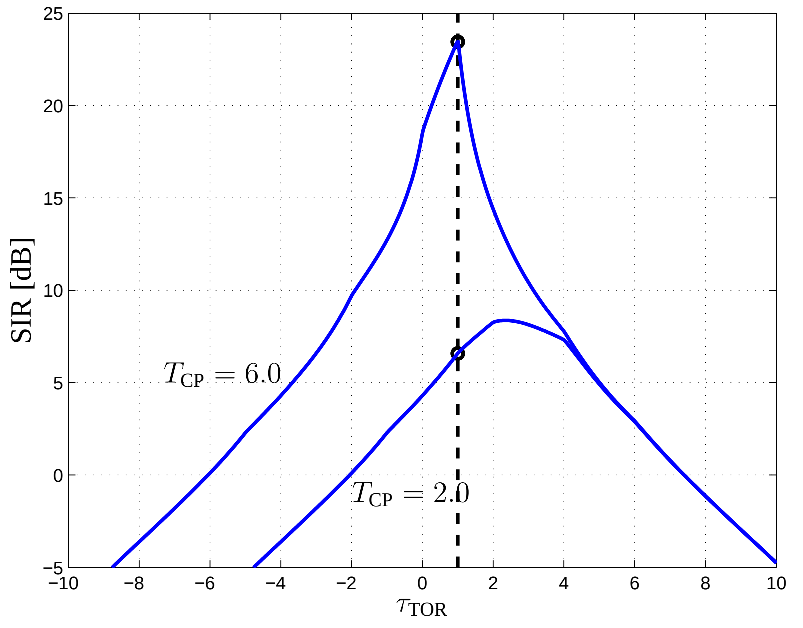
Time Synchronization in OFDM (4)



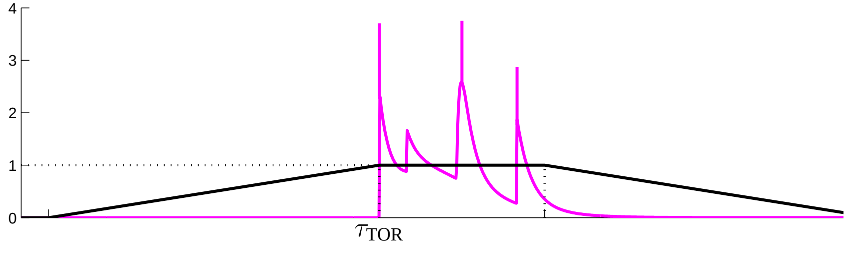
- Synchronization to the first cluster is almost optimal ($\tau_{\text{TOR}} = 1.0$):

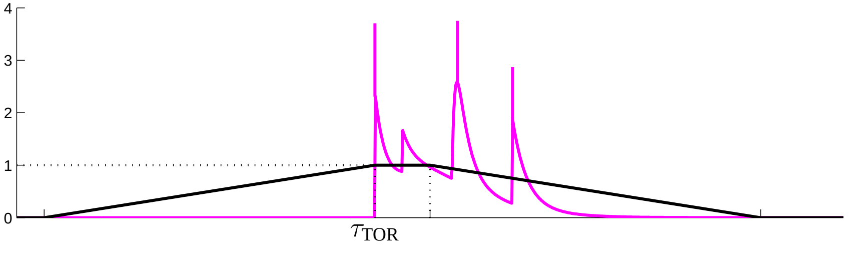


Physical Layer Parameters in OFDM



- Longer and shorter cyclic prefix ($T_{\text{CP}} = 4.0$ in previous slides):





Conclusion

Conclusion

- A generalized power–delay profile for multipath channels
  - Sum of exponentially-decaying clusters
  - Line-of-sight path, specular reflections, diffraction, diffuse reflections with single and multiple scattering
- Characterization of propagation environment
  - Analytical evaluation of the performance of OFDM transmission through the modeled channel
    - Subcarrier spacing, pilot structures
    - Non-ideal time synchronization
    - Physical layer parameters

The End

"You can always find an environment that fits your model."
— Prof. Jørgen Bach Andersen[1]

[1] Citation adopted from V.-M. Kolmonen, D.Sc. thesis, Aalto University, April 2010

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