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RE11. On Signal Bandwidths in Cooperative Communications

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Introduction

- The $(1 \times N_\mathrm{r} \times 1)$ relay network model: The source broadcasts, the relays amplify-and-forward (AF) and the destination receives a superposition:

$$r_k(\tau) = \sqrt{E_{\mathrm{S}k}} h_{\mathrm{S}k} x(\tau - \tau_{\mathrm{S}k}) + n_k \quad (1)$$
$$t_k(\tau) = \beta_k r_k(\tau) \quad (2)$$
$$y(\tau) = \sum_{k=1}^{N_\mathrm{r}} \sqrt{E_{k\mathrm{D}}} h_{k\mathrm{D}} t_k(\tau - \tau_{k\mathrm{D}}) + n_\mathrm{D} \quad (3)$$

- Previous research often assumes that the relayed signals arrive synchronously at the receiver.
- In practical wireless networks, spatial separation causes different propagation delays for the signal components and the destination observes a multipath channel.

Narrowband Reception

- Assuming the relays are uniformly distributed with density ρ, inherent delay spread is created.

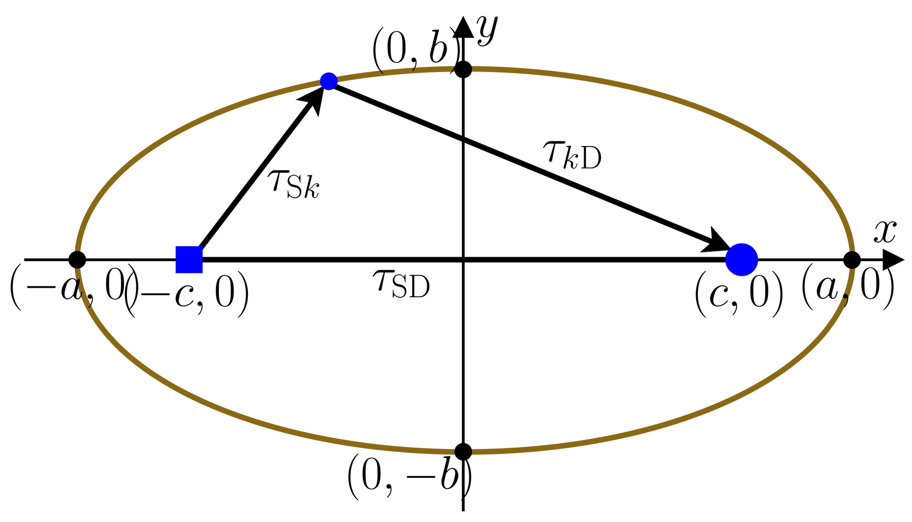

Fig. 1: Elliptical geometry for relays producing equal delays.

- To minimize the maximum delay τ_max, the relays need to be selected inside an ellipse that should be large enough for accomodating N_r relays, i.e., we obtain

$$\tau_\mathrm{max} \geq \frac{1}{v}\sqrt{2c^2 + 2\sqrt{c^4 + 4\left(\frac{N_\mathrm{r}}{\rho\pi}\right)^2}}. \quad (4)$$

- For narrowband reception, the maximum delay τ_max determines also the maximum available bandwidth

$$W \leq \frac{\delta v}{\sqrt{2c^2 + 2\sqrt{c^4 + 4\left(\frac{N_\mathrm{r}}{\rho\pi}\right)^2}} - 2c} = \mathcal{O}\left(\frac{1}{\sqrt{N_\mathrm{r}}}\right). \quad (5)$$

- Fig. 2 provides an example of available bandwidths for varying relay densities and number of relays when c=100 m and δ=0.1.

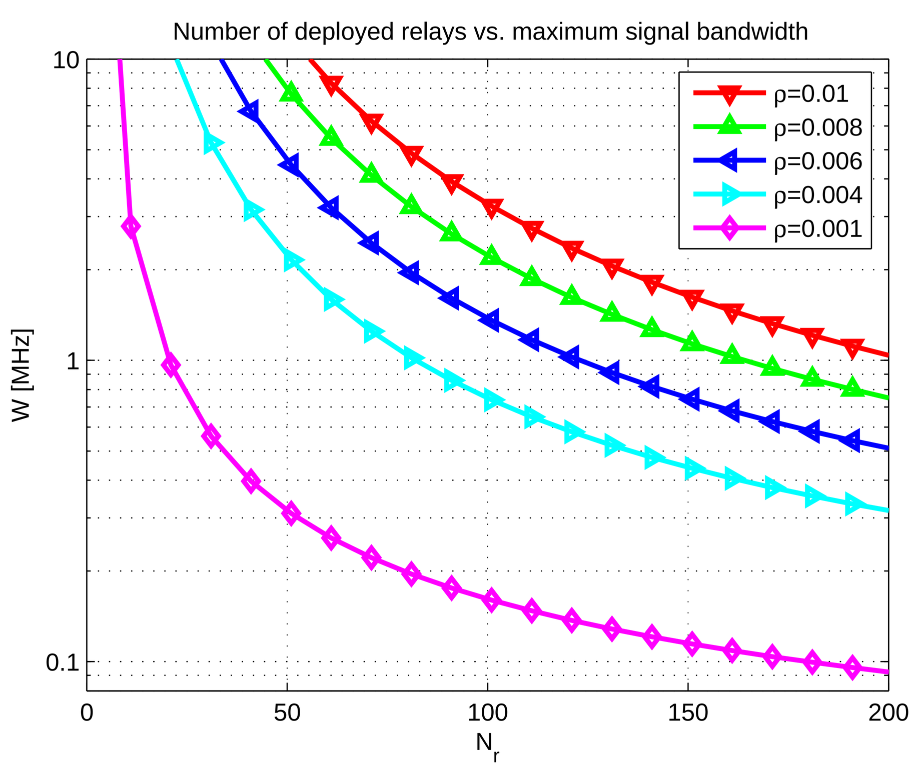

Fig. 2: Maximum available bandwidth in narrowband reception.

Wideband Reception

- If the cascade of transmit and receive filters is an ideal low-pass filter with a cut-off frequency $W/2$, the received energy via the multipath channel created by the relays is

$$E_\mathrm{rcv} = \frac{1}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}} |H(f)|^2 \mathrm{d}f. \quad (6)$$

- The diversity metric

$$\Delta = \frac{\mathrm{Var}\left[E_\mathrm{rcv}\right]}{\left(\mathcal{E}\left[E_\mathrm{rcv}\right]\right)^2} \quad (7)$$

is studied as a function of signal bandwidth.

- Bounds for the diversity metric:

$$\alpha \leq \Delta \leq \beta/W^2 + \alpha, \quad (8)$$

where α arises from the irreducible variances of the channel taps and β is a constant depending on the network geometry.

  - By increasing system bandwidth, it is possible to resolve more relay channels which improves diversity. However, increasing bandwidth beyond a certain value does not give any benefits, because the diversity metric converges to a floor value.
  - For fixed amplitude channels $\alpha = 0$ and the diversity metric can achieve arbitrary small values.
  - Minimal value for β is achieved with uniform delay distribution.
  - The floor value without CSI (double-Rayleigh channels) is 4.77 dB higher compared to the situation with receive CSI (Rayleigh channels).

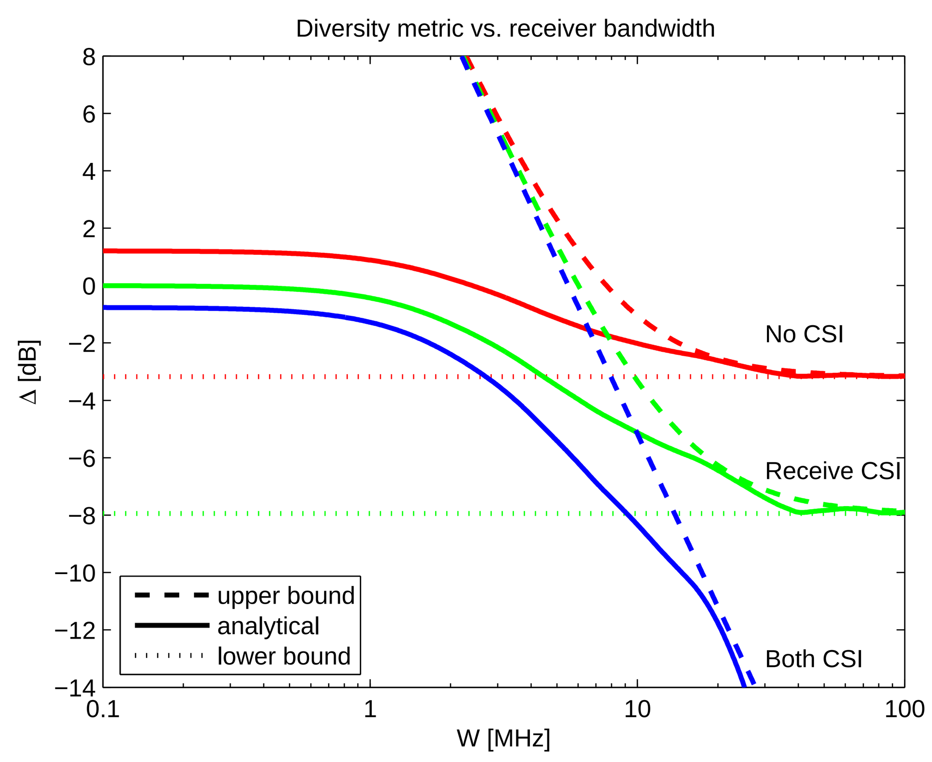

Fig. 3: Diversity behaviour of an example network.

Conclusion

- Gaussian parallel relay networks are narrowband in nature.
- When performing narrowband reception, the maximum available bandwidth is reached, when the N_r uniformly distributed relays are located inside an ellipse.
- When performing wideband reception, diversity improves with increasing bandwidth up to a “knee bandwidth” and then converges to a floor value
- Only if the relays were able to exploit full channel state information and invert end-to-end channels perfectly, the metric would be arbitrarily small.
- We see future research in developing relaying protocols that take into account delay spread, transceiver dynamics, SNR and power constraints.

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