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Transmission Rate Performance of Symmetric Two-Way Full-Duplex Links at Large Antenna-Array Limit

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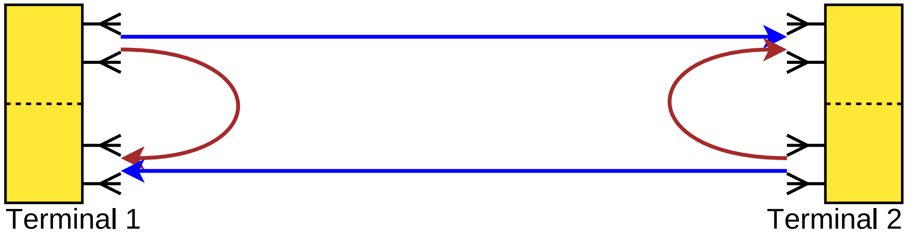
Introduction

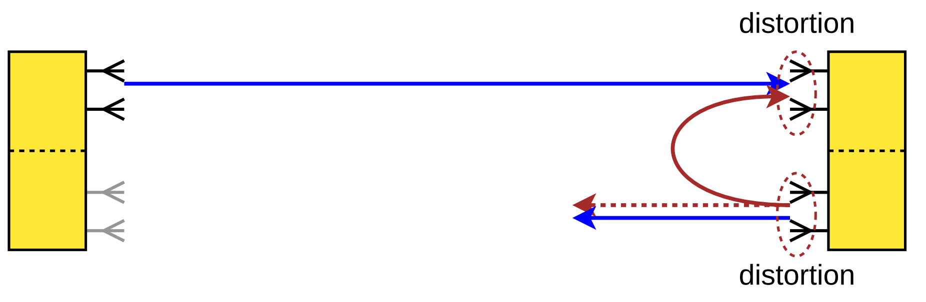
In-band Full-Duplex Wireless Point-to-Point Links



- Two-way transmission without the allocation of extra channel resources
  - ▷ efficient frequency reuse
  - ▷ loopback self-interference!

- *Transmission rates* for practical *digital modulation* (e.g., PSK or QAM)
  - ▷ Preliminary numerical results on symmetric scenarios
- *The replica method*: originally developed in the field of statistical physics and recently applied to communication theory problems
  - ▷ the *large-system limit* assumption
  - ▷ Simulated results for small-scale systems agree well with the corresponding asymptotic results

Non-ideal Electronics and Mismatched Decoding



- Rx: imperfect conversion from analog RF to digital baseband
- Tx: imperfect conversion from digital baseband to analog RF plus nonlinear amplification

- *Feedback* transmit-side noise may be on a par with the far-end signal due to the high gain of the near-end interference channel
  - ▷ *Feedforward* transmit-side noise can be neglected since it is typically below receive-side noise after channel attenuation
- Mitigation transparently around the actual multiplexing protocol which can operate without being aware of self-interference
  - ▷ *Mismatched* detection and decoding due to unexpected noise

System Model

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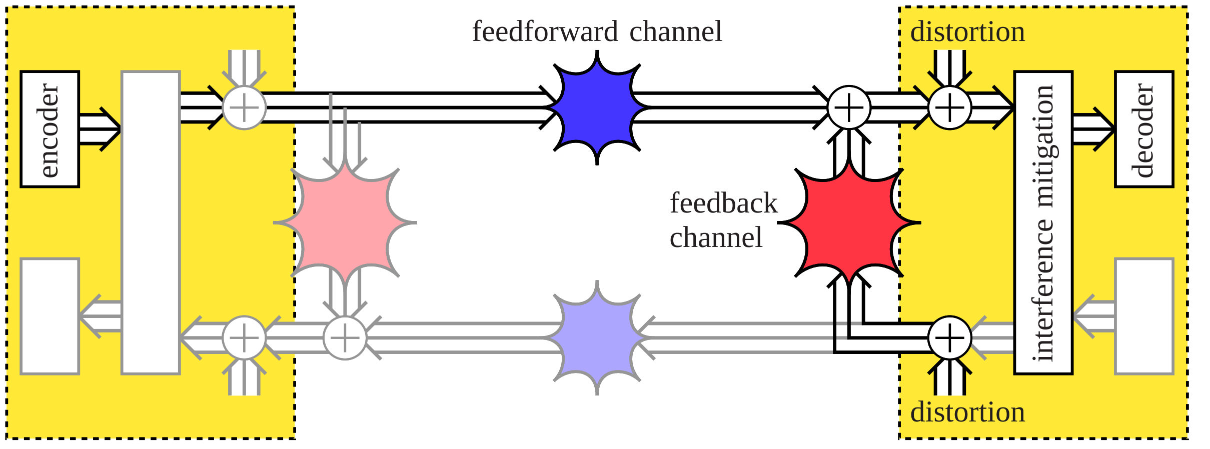
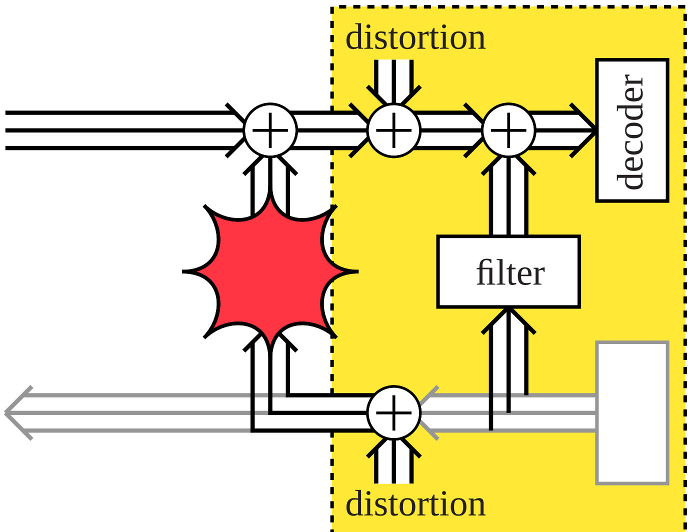


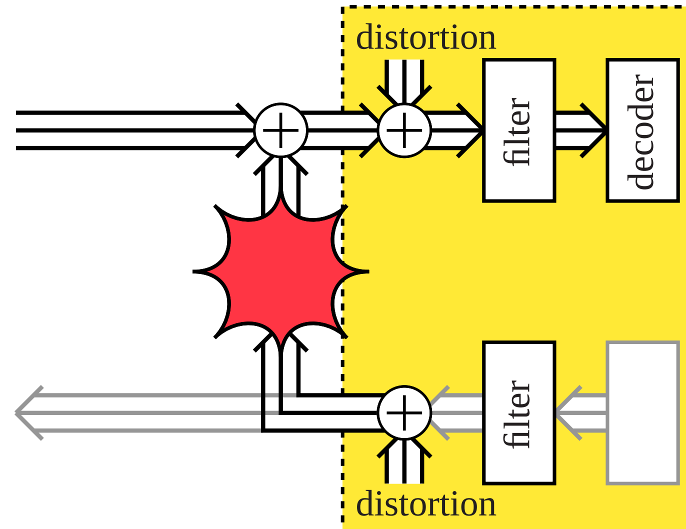
- Front ends: self-interference mitigation based on a hybrid of time-domain cancellation and spatial-domain suppression
- Back ends: regular digital open-loop spatial multiplexing
  - separate encoding of QPSK, 8-PSK, or 16-QAM
  - mismatched joint decoding despite perfect CSI
- Feedforward channels are i.i.d. complex Gaussian matrices while feedback channels are zero-mean and i.i.d. with any distribution

Self-interference Mitigation

Time-domain cancellation:



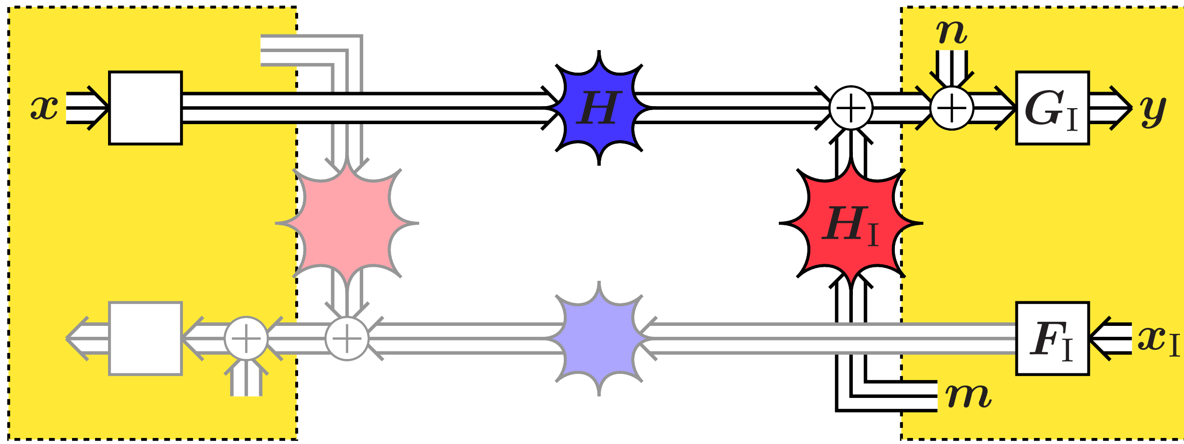
Spatial-domain suppression:



The order adopted herein:

1. Cancellation subtracts away the known part of self-interference
2. Rx-side suppression for reducing the effect of Tx-side distortion
3. Tx-side suppression for alleviating Rx-side dynamic-range issues

Signal Model



- Each terminal has M transmit antennas and N receive antennas
  - $\hat{M}$ transmit streams and $\hat{N}$ receive streams after suppression
- Effective post-processing signal model after mitigation:

$$\boldsymbol{y} = \hat{\boldsymbol{H}}\boldsymbol{x} + \boldsymbol{w} \quad \text{where} \quad \boldsymbol{w} = \boldsymbol{G}_{\mathrm{I}}(\boldsymbol{H}_{\mathrm{I}}\boldsymbol{m} + \boldsymbol{n})$$

  - noise terms $\boldsymbol{m}$ and $\boldsymbol{n}$ due to transceiver imperfections:
    $\sigma_{\boldsymbol{m}}^2 = (\sigma_0^2 + \rho_{\boldsymbol{m}} p_{\mathrm{I}})/M$ and $\sigma_{\boldsymbol{n}}^2 = \sigma_0^2 + \rho_{\boldsymbol{n}}(pg + p_{\mathrm{I}}\|\boldsymbol{H}_{\mathrm{I}}\boldsymbol{F}_{\mathrm{I}}\|_{\mathrm{F}}^2/(\hat{M}N))$

Analytical Results

Problem Formulation

- The true covariance matrix of residual distortion and noise after mitigation: $\boldsymbol{R}_{\boldsymbol{w}} = \mathcal{E}\{\boldsymbol{w}\boldsymbol{w}^{\mathrm{H}}\} = \sigma_{\boldsymbol{m}}^2 \boldsymbol{G}_{\mathrm{I}} \boldsymbol{H}_{\mathrm{I}} \boldsymbol{H}_{\mathrm{I}}^{\mathrm{H}} \boldsymbol{G}_{\mathrm{I}}^{\mathrm{H}} + \sigma_{\boldsymbol{n}}^2 \boldsymbol{G}_{\mathrm{I}} \boldsymbol{G}_{\mathrm{I}}^{\mathrm{H}}$
  - ▹ Mismatched decoding estimates $\boldsymbol{R}_{\boldsymbol{w}}$ as some fixed $\tilde{\boldsymbol{R}}_{\boldsymbol{w}}$ and uses a postulated density $\tilde{f}(\boldsymbol{y} \mid \boldsymbol{x}, \hat{\boldsymbol{H}}) = g(\boldsymbol{y} \mid \hat{\boldsymbol{H}}\boldsymbol{x};\, \tilde{\boldsymbol{R}}_{\boldsymbol{w}})$
- Generalized mutual information (GMI) is defined as

$$I_{\mathrm{GMI}}(\boldsymbol{y};\, \boldsymbol{x}) = \sup_{s>0} I_{\mathrm{GMI}}^{(s)}(\boldsymbol{y};\, \boldsymbol{x}) \text{ where } I_{\mathrm{GMI}}^{(s)}(\boldsymbol{y};\, \boldsymbol{x}) = \mathcal{E}\left\{ \ln\left(\frac{\tilde{f}(\boldsymbol{y} \mid \boldsymbol{x}, \boldsymbol{H})^s}{\mathcal{E}_{\boldsymbol{x}}\{\tilde{f}(\boldsymbol{y} \mid \boldsymbol{x}, \boldsymbol{H})^s\}} \right) \right\}$$

  - ▹ For technical reasons, we set $s = 1$ yielding a lower bound
- We use *the replica method* instead of directly evaluating

$$I_{\mathrm{GMI}}^{(s)}(\boldsymbol{y};\, \boldsymbol{x}) = \hat{M} \ln \kappa - s\, \mathrm{tr}\{\tilde{\boldsymbol{R}}_{\boldsymbol{w}}^{-1} \boldsymbol{R}_{\boldsymbol{w}}\}$$
$$- \frac{1}{\kappa^{\hat{M}}} \sum_{\boldsymbol{x}} \mathcal{E}_{\boldsymbol{H}_{\mathrm{I}}, \hat{\boldsymbol{H}}} \left\{ \int_{\mathbb{C}^{\hat{N}}} g(\boldsymbol{w} \mid \boldsymbol{0};\, \boldsymbol{R}_{\boldsymbol{w}}) \ln\left(\sum_{\tilde{\boldsymbol{x}}} e^{-[\hat{\boldsymbol{H}}(\boldsymbol{x}-\tilde{\boldsymbol{x}})+\boldsymbol{w}]^{\mathrm{H}} s \tilde{\boldsymbol{R}}_{\boldsymbol{w}}^{-1} [\hat{\boldsymbol{H}}(\boldsymbol{x}-\tilde{\boldsymbol{x}})+\boldsymbol{w}]} \right) \mathrm{d}\boldsymbol{w} \right\}$$

Proposition

- The set of constellation points of i.i.d. transmit symbols: $\{\sigma_{\boldsymbol{x}} \chi_k\}_{k=1}^{\kappa}$
- Let the postulated noise covariance $\tilde{\boldsymbol{R}}_{\boldsymbol{w}}$ be a scaled identity matrix
- A lower bound to the maximum *per-transmit-antenna achievable rate* [nat/s/Hz] is given by the replica method as

$$\boxed{R = \hat{m}\left(I(z;\,\chi) - \frac{\ln(\eta\,\bar{\sigma}_{\boldsymbol{w}}^2) + pg\,\eta\,\varepsilon}{\hat{\alpha}}\right)}$$

where $\hat{\alpha} = \hat{M}/\hat{N}$, $\bar{\sigma}_{\boldsymbol{w}}^2 = \mathrm{tr}\{\boldsymbol{R}_{\boldsymbol{w}}\}/\hat{N}$,

$$I(z;\,\chi) = -1 - \ln\hat{\alpha} - \ln\frac{\pi}{\eta} - \int_{\mathbb{C}} f(z)\ln f(z)\,\mathrm{d}z$$

$$f(z) = \frac{1}{\kappa}\sum_{k=1}^{\kappa} g(z \mid \sqrt{pg}\,\chi_k;\,\hat{\alpha}\,\eta^{-1})$$

and the remaining parameters are given by

$$\eta = \frac{1}{pg\,\varepsilon + \bar{\sigma}_{\boldsymbol{w}}^2}, \quad \varepsilon = 1 - \int_{\mathbb{C}} \frac{1}{f(z)}\left|\frac{1}{\kappa}\sum_{k=1}^{\kappa}\chi_k\,g(z \mid \sqrt{pg}\,\chi_k;\,\hat{\alpha}\,\eta^{-1})\right|^2 \mathrm{d}z$$

Remarks

- The achievable rate obtained in the Proposition (previous slide) does not depend on the postulated noise variance
  - ▷ Nearest neighborhood decoding is sufficient
- The replica calculation fixes also the parameter s, so that the supremum over $s > 0$ is not part of the achievable rate calculation
  - ▷ to guarantee a stable replica symmetric solution
  - ▷ not necessarily the best lower bound of this form
  - ▷ The structure of the solution strongly suggests that the stable solution given by the replica method is indeed the supremum or at least very close to it
- Relatively low computational complexity compared to Monte Carlo simulations of exponential complexity

Numerical Results

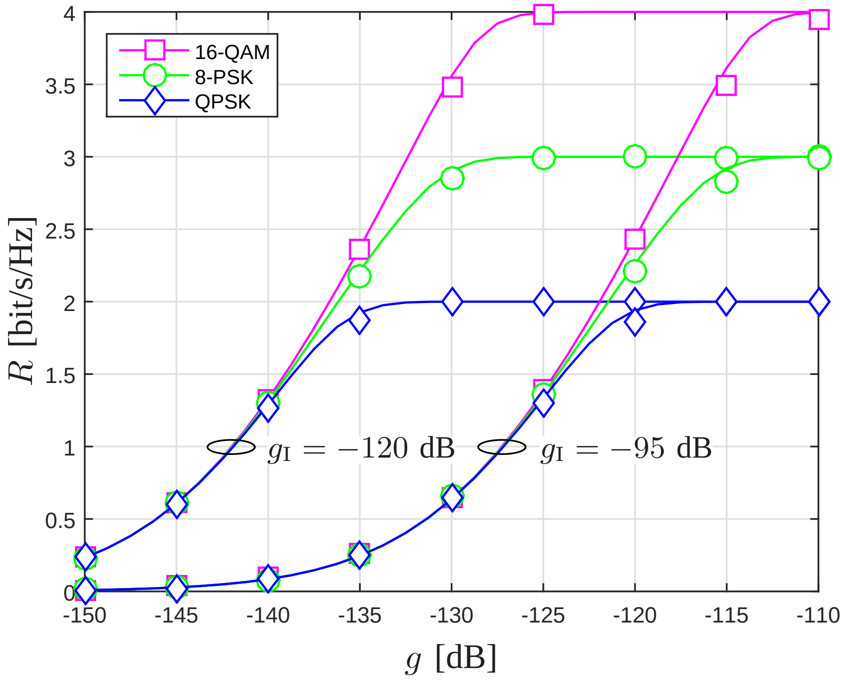
Example Setup

- Symmetric systems where
  - $p = p_{\text{I}} = 36$ dBm
  - $\sigma_0^2 = -104$ dBm
  - $\rho_{\boldsymbol{m}} = -30$ dB
  - $\rho_{\boldsymbol{n}} = -60$ dB
- Per-antenna transmission rates for QPSK, 8-PSK, and 16-QAM
- There are five key system parameters to explore:

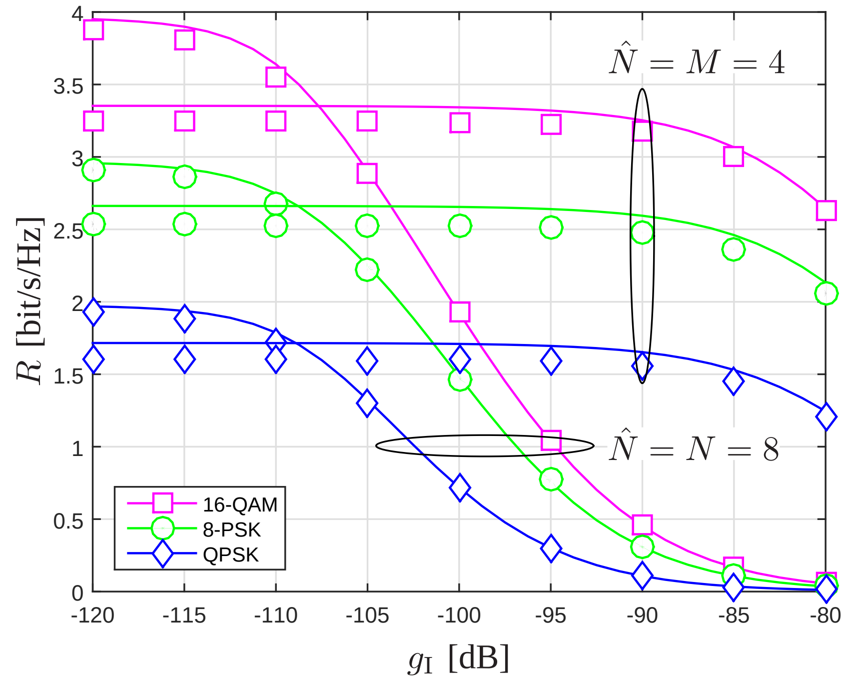
$$\boxed{g} \quad \boxed{g_{\text{I}}} \quad \boxed{\alpha = M/N} \quad \boxed{\hat{m} = \hat{M}/M} \quad \boxed{\hat{n} = \hat{N}/N}$$

  - These remain constants when M and N grow asymptotically
- Extremely time-consuming Monte Carlo computations for a finite-sized reference system with $M = 4$ and $N = 8$

Transmission Rates vs. Channel Gains



(a) $g_{\mathrm{I}} = \{-120, -95\}$ dB, $\hat{N} = N = 8$



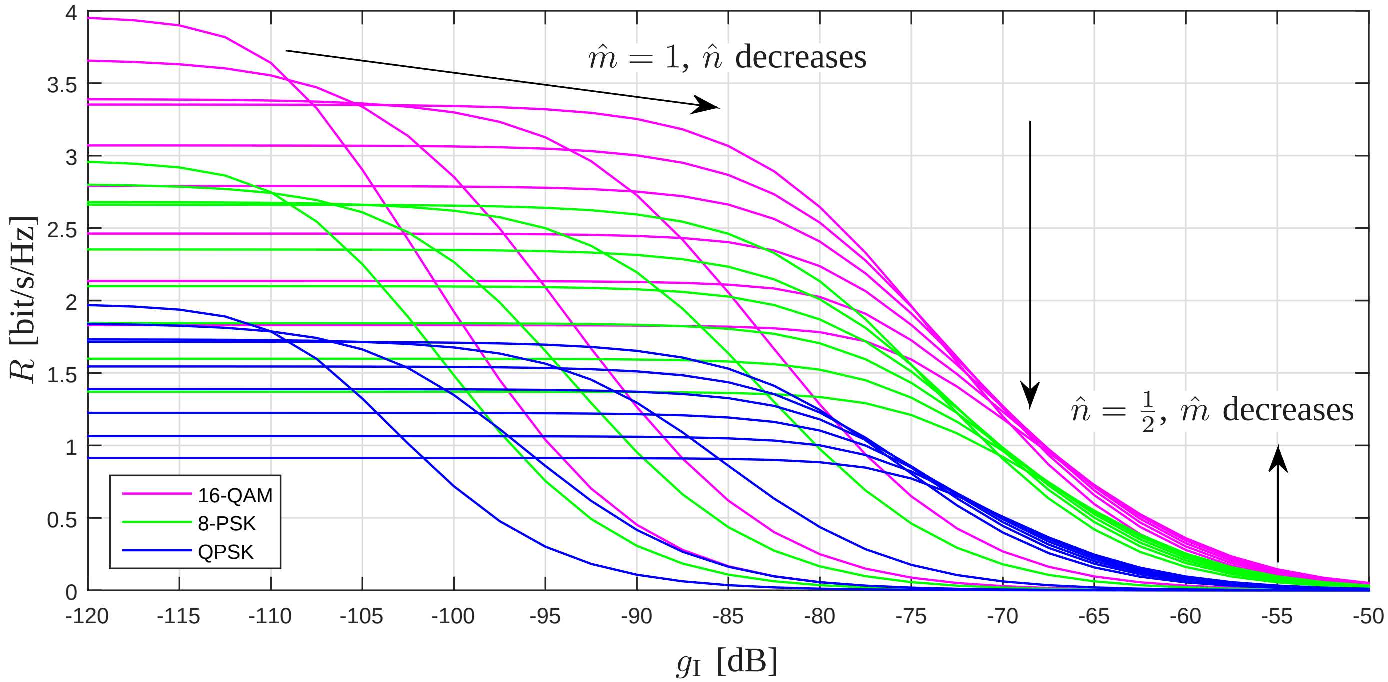
(b) $g = \{-134, -129, -127\}$ dB

- Simulations (markers) corroborate analytical results (solid lines)

(a) increasing self-interference causes a shift in the target SNR range

(b) Rx-side suppression eliminates Tx-side noise when $\hat{N} \leq N - M$

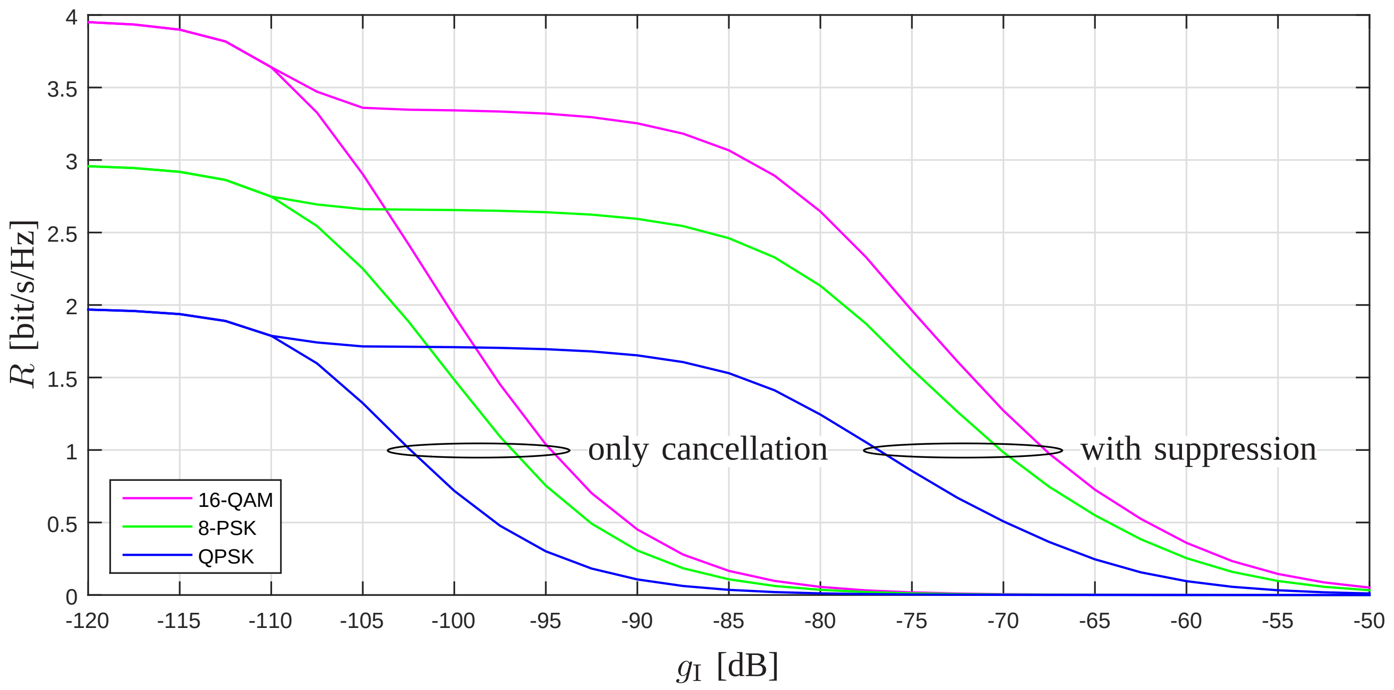
Optimization of Stream Configuration



$$g = \{-134\ (\text{QPSK}), -129\ (\text{8-PSK}), -127\ (\text{16-QAM})\}\ \text{dB}$$

- Low g_{I}: use all degrees of freedom for spatial multiplexing
- Medium g_{I}: reduce the number of Rx-streams
- High g_{I}: reduce also the number of Tx-streams

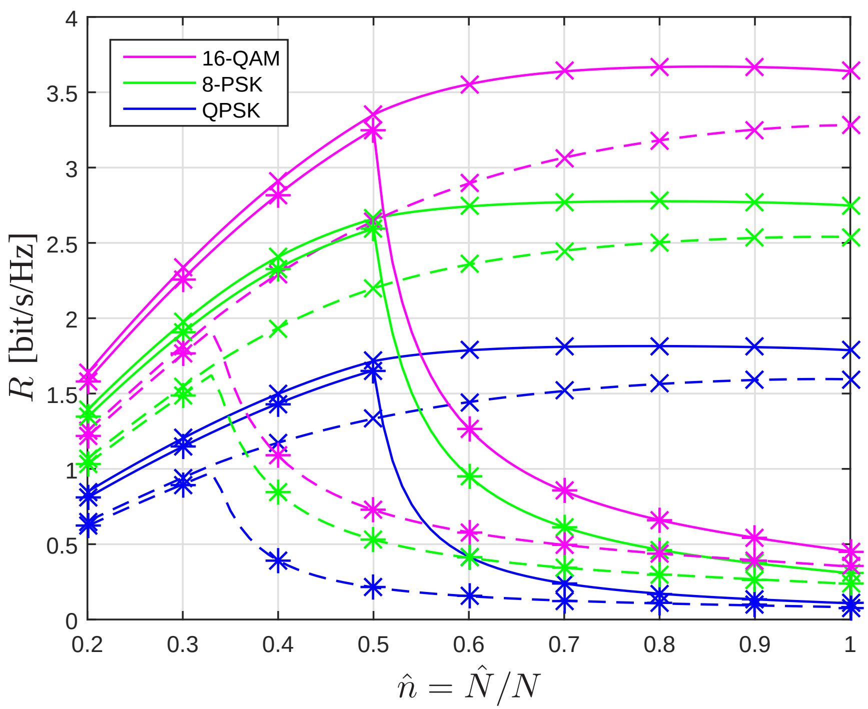
Gain from Suppression after Cancellation



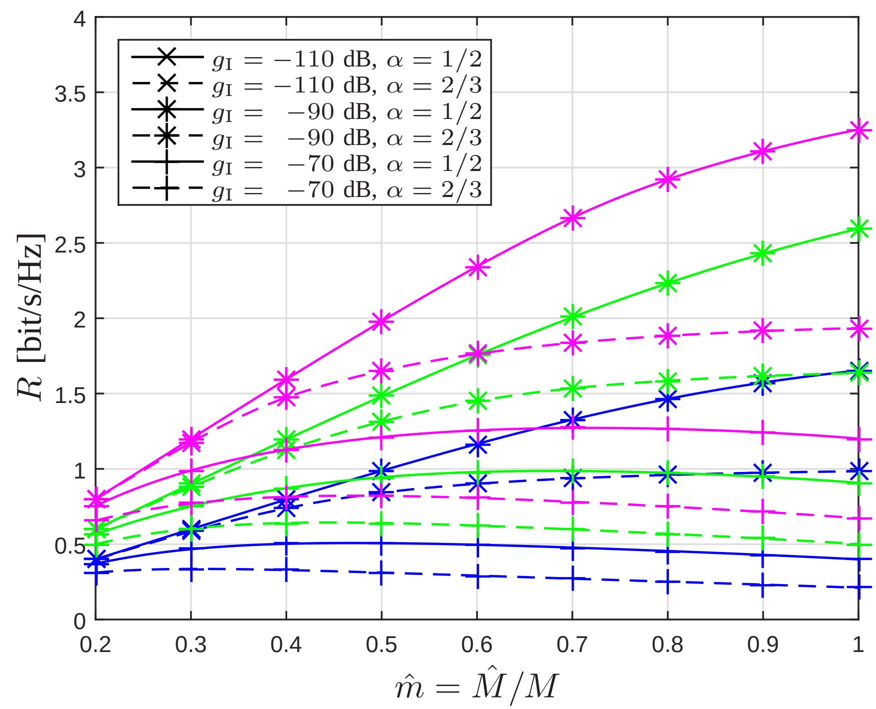
$$g = \{-134\ (\text{QPSK}), -129\ (\text{8-PSK}), -127\ (\text{16-QAM})\}\ \text{dB}$$

- It is very beneficial to implement spatial-domain suppression in addition to time-domain cancellation unless physical isolation is large and mitigation is almost not needed at all

Transmission Rates vs. Stream Configurations



(a) $g_{\mathrm{I}} \in \{-110, -90\}$ dB, $\hat{m} = \hat{M}/M = 1$



(b) $g_{\mathrm{I}} \in \{-90, -70\}$ dB, $\hat{n} = \hat{N}/N = 1 - \alpha$

- Sensible range: $\hat{n} \geq \max\{0, 1-\alpha\}$ and $\hat{m} \geq \max\{0, 1-\alpha^{-1}\}$

(a) Rx-side suppression by decreasing $\hat{n}$ yields the largest gain

(b) decreasing $\hat{m}$ affects directly the multiplexing order of the system

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Conclusion

Conclusion

- Analysis of two-way point-to-point in-band full-duplex MIMO links
  - Related works: ideal Gaussian encoding and matched decoding
  - This study: practical *digital modulation* and non-ideal *mismatched joint decoding* (due to residual self-interference)
- A lower bound for the achievable transmission rate
  - Scope limitation: symmetric antenna configurations
- By optimizing stream configurations, the link may control the trade-off between spatial multiplexing and self-interference suppression implemented in addition to subtractive cancellation
- Reference simulations showed that the theoretical large-array results are convenient approximations for realistic finite-sized arrays

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