WC-I04 Asymptotic Analysis of Asymmetric MIMO Links: EVM Limits for Joint Decoding of PSK and QAM

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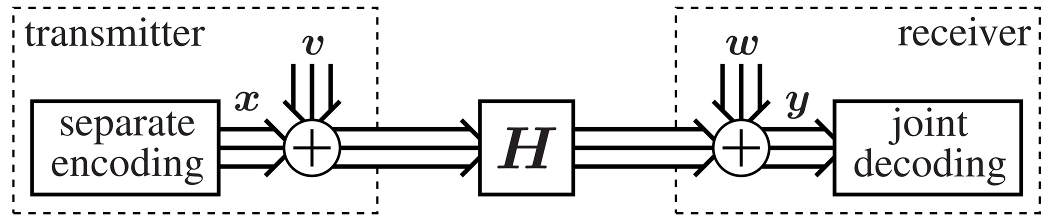
Summary

Hardware non-idealities in wireless transmitter electronics cause distortion that is not captured by conventional linear channel models. Motivated by *error-vector magnitude (EVM)* measurements in conformance testing, herein the achievable rate of a 'binoisy' multiple-input multiple-output (MIMO) channel

$$\boldsymbol{y} = \underbrace{\boldsymbol{H}}_{M\times N}(\boldsymbol{x}+\boldsymbol{v}) + \boldsymbol{w} \in \mathbb{C}^M, \tag{1}$$

is considered. The non-idealities manifest themselves as an additive noise term $\boldsymbol{v} \in \mathbb{C}^N$ at the transmit side. *Large system analysis* covering both Gaussian and *practical digital modulation* schemes is presented and numerical results illustrate how *tolerable EVM levels* depend non-trivially on various factors, such as, signal-to-noise ratio, modulation order and the level of asymmetry in antenna array configurations.

System Model



The system model related to the received signal given by (1) is depicted in the figure above. Transmitter is assumed to use *spatial multiplexing* and the receiver knows the PDFs of the noise plus distortion terms $\boldsymbol{v}$ and $\boldsymbol{w}$ as well as the distribution of the data vector $\boldsymbol{x}$. The conditional PDF

$$p(\boldsymbol{y} \mid \boldsymbol{x}, \boldsymbol{H}) = g(\boldsymbol{y} \mid \boldsymbol{H}\boldsymbol{x};\ \boldsymbol{R}_{\boldsymbol{w}} + \boldsymbol{H}\boldsymbol{R}_{\boldsymbol{v}}\boldsymbol{H}) \tag{2}$$

is used for *matched joint decoding* of the transmitted signals.

Here $g(\boldsymbol{y} \mid \boldsymbol{m};\ \boldsymbol{R}_{\boldsymbol{y}}) = \pi^{-M}\det(\boldsymbol{R}_{\boldsymbol{y}}^{-1})\exp[-(\boldsymbol{y}-\boldsymbol{m})^{\mathsf{H}}\boldsymbol{R}_{\boldsymbol{y}}^{-1}(\boldsymbol{y}-\boldsymbol{m})]$, denotes the proper complex Gaussian density with mean $\boldsymbol{m}$ and covariance matrix $\boldsymbol{R}_{\boldsymbol{y}}$.

Achievable Rate

In the ideal case, codewords span infinitely many independent channel realizations and the *achievable rate* is given by the ergodic mutual information (in nats):

$$\begin{aligned} I(\boldsymbol{y};\boldsymbol{x}) &= \overbrace{-\mathsf{E}\{\ln \mathsf{E}_{\boldsymbol{x}}\{p(\boldsymbol{y} \mid \boldsymbol{x}, \boldsymbol{H})\}\}}^{=h(\boldsymbol{y})} + \overbrace{\mathsf{E}\{\ln p(\boldsymbol{y} \mid \boldsymbol{x}, \boldsymbol{H})\}}^{=-h(\boldsymbol{y}|\boldsymbol{x})} \\ &= h(\boldsymbol{y}) - h(\boldsymbol{y} \mid \boldsymbol{x}), \end{aligned} \tag{3}$$

where the outer expectations are w.r.t. all random varibles in (1). The achievable rate $I(\boldsymbol{y};\boldsymbol{x})$ was investigated in the case of Gaussian signaling in [1]. However, the case of practical digital modulation such as PSK and QAM has remained an open problem.

Main goal of the paper: Evaluate (3) for *PSK and QAM* channel inputs.

GOAL: Evaluate the Achievable Rate in (3)

1) From (2) we get $h(\boldsymbol{y} \mid \boldsymbol{x}) = \mathsf{E}_{\boldsymbol{H}}\{\ln\det(\boldsymbol{R}_{\boldsymbol{w}} + \boldsymbol{H}\boldsymbol{R}_{\boldsymbol{v}}\boldsymbol{H}^{\mathsf{H}})\} + N\ln\pi + N$.

- Expectation over channel can be evaluated, e.g., using MC methods or random matrix theory (RMT) $\implies$ $h(\boldsymbol{y} \mid \boldsymbol{x})$ is "easy" to compute.

2) To obtain an expression for $h(\boldsymbol{y})$, we need to evaluate a term:

$$\sum_{\boldsymbol{x}\in\mathcal{A}^M} \mathsf{E}_{\boldsymbol{v},\boldsymbol{w},\boldsymbol{H}}\left\{\ln\left(\sum_{\boldsymbol{x}'\in\mathcal{A}^M} \mathrm{e}^{-[\boldsymbol{H}(\boldsymbol{x}-\boldsymbol{x}'+\boldsymbol{v})+\boldsymbol{w}]^{\mathsf{H}}(\boldsymbol{R}_{\boldsymbol{w}}+\boldsymbol{H}\boldsymbol{R}_{\boldsymbol{v}}\boldsymbol{H})^{-1}[\boldsymbol{H}(\boldsymbol{x}-\boldsymbol{x}'+\boldsymbol{v})+\boldsymbol{w}]}\right)\right\}$$

where $\mathcal{A}$ is the modulation set (for example, PSK or QAM).

- MC computation has *exponential complexity* and *RMT does not work*.
- The term $h(\boldsymbol{y})$ seems intractable for conventional methods.

Solution: Use the (non-rigorous) *replica method* from statistical physics.

Result

Consider the simplified case $\boldsymbol{R}_{\boldsymbol{v}} = r_{\boldsymbol{v}}\boldsymbol{I}$ and $\boldsymbol{R}_{\boldsymbol{w}} = r_{\boldsymbol{w}}\boldsymbol{I}$, i.e., the antennas experience spatially white distortion at both ends of the link. Then, for large M, N, the ergodic MI of the original MIMO system (3) can be approximated as

$$\boxed{I(\boldsymbol{y};\boldsymbol{x}) = I(z;\chi) + \ln\left(\frac{r_{\boldsymbol{w}}+\varepsilon}{r_{\boldsymbol{w}}+\varepsilon'}\right) - \ln(1+\eta' r_{\boldsymbol{v}}) - \eta\varepsilon + \eta'\varepsilon',} \tag{4}$$

where η, ε (similarly η', ε') are solutions to coupled fixed point equations and $I(z;\chi)$ is the MI of a scalar AWGN channel (see [2] and the paper for extensions and details).

Conclusion: The analysis of a *fading MIMO channel reduces* to analysis of an equivalent *non-fading SISO* channel!

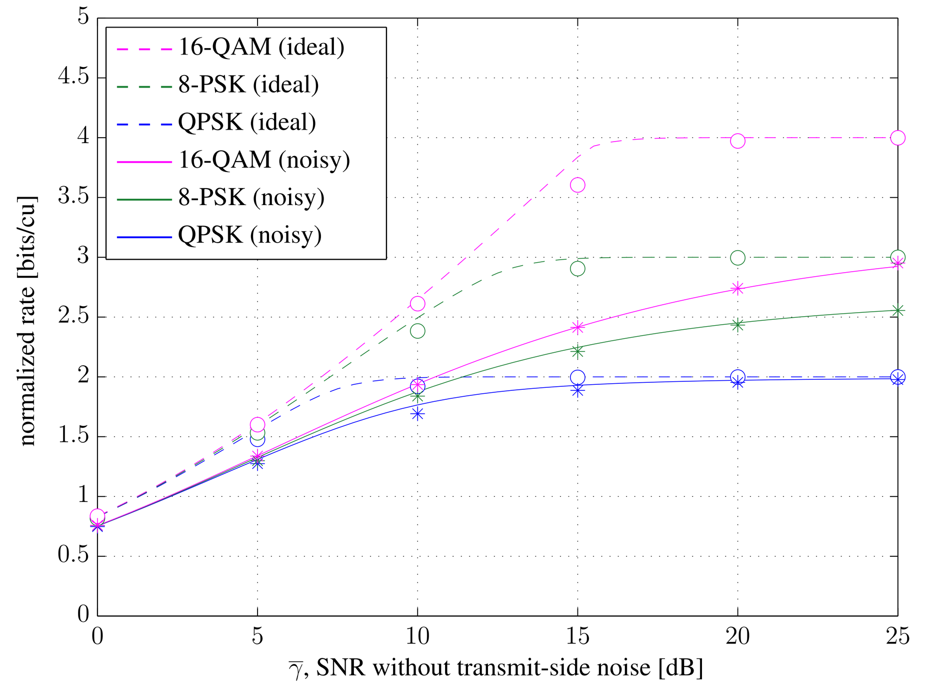
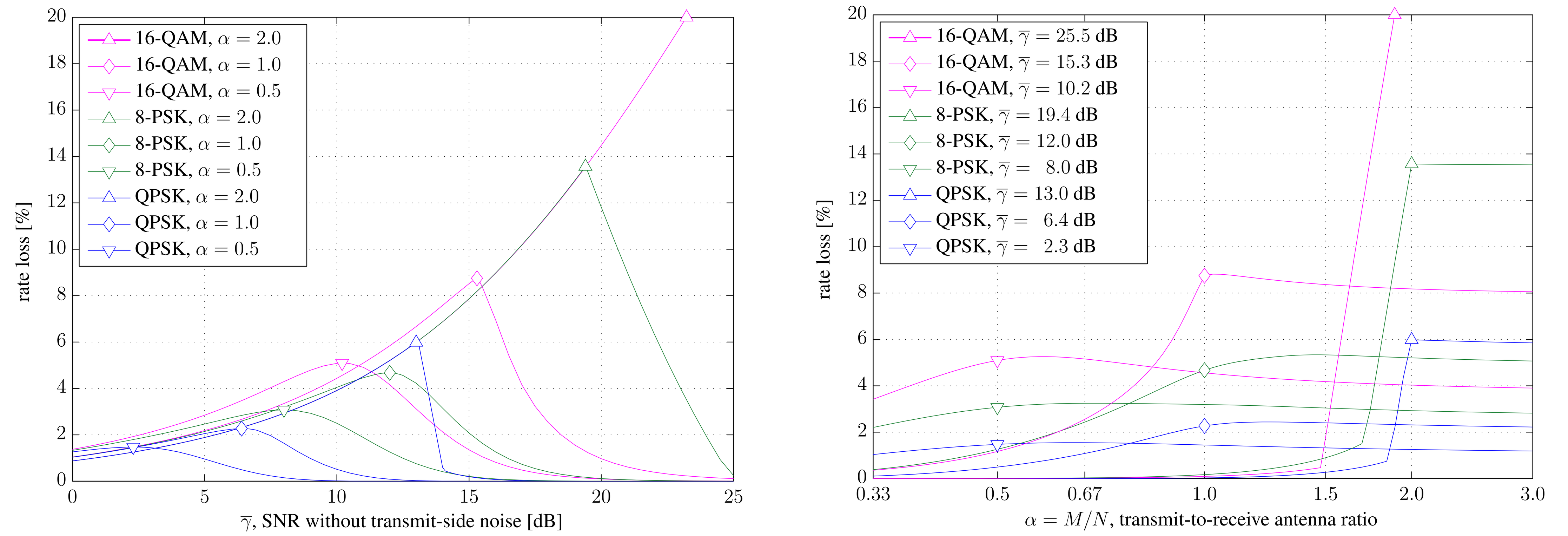


Figure: Normalized rate $M^{-1}I(\boldsymbol{y};\boldsymbol{x})$ with ideal ($\mathsf{EVM} = -\infty$ dB) and non-ideal ($\mathsf{EVM} = -10$ dB) hardware. Markers for MC simulations with $M = N = 4$.

Numerical Examples



(a) Rate loss vs. SNR for antenna ratios $\alpha = M/N \in \{1/2, 1, 2\}$. Markers depict the maximum rate loss (in percentage) for each of the cases within the given SNR region.

(b) Rate loss vs. antenna ratios $\alpha = M/N$. The SNRs match markers on left, except for the high-SNR case of 16-QAM that corresponds to maximum rate loss at $\alpha = 2$.

Figure: Rate loss percentage for noisy setup ($\mathsf{EVM} = -20$ dB) when compared to ideal hardware.

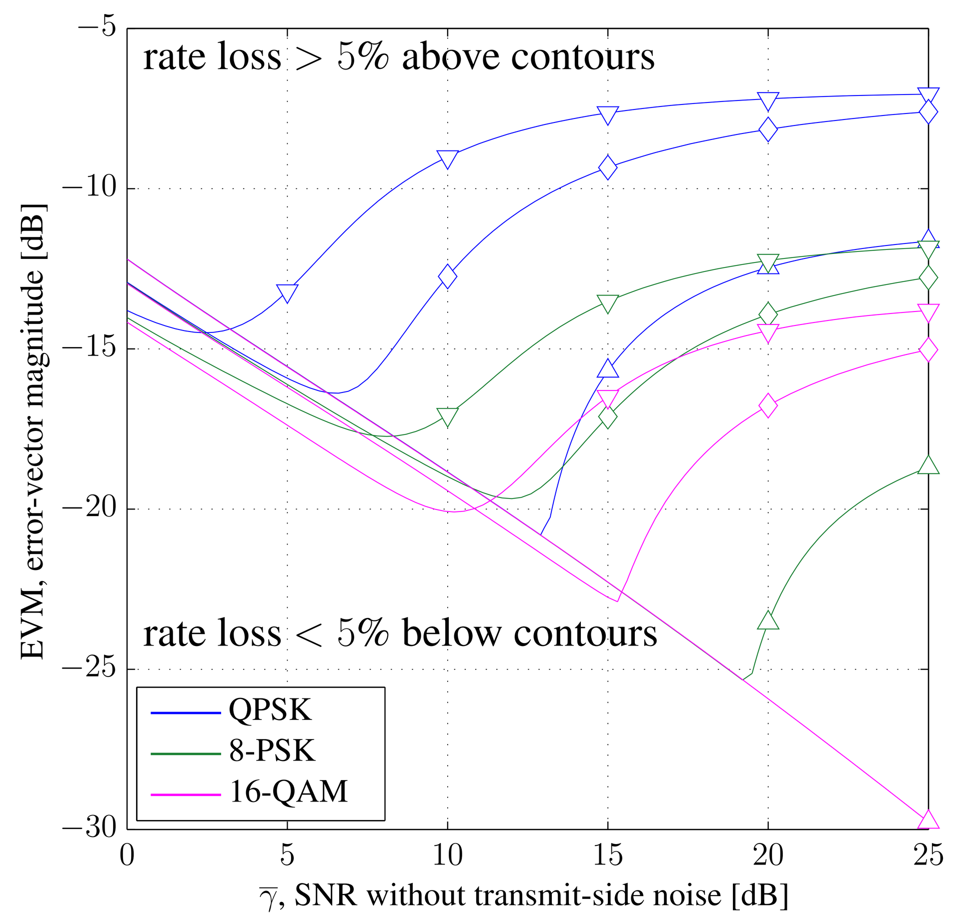


Figure: The 5% rateloss contours for antenna ratios $\alpha \in \{1/2, 1, 2\}$. The regions above curves define $(\bar{\gamma}, \mathsf{EVM})$ pairs for rate losses higher than 5%, and vice versa for the areas below the curves.

EVM Target for PSK and QAM

If the EVM remains below the values tabulated in the table, hardware non-idealities can be considered negligible.

	transmit-to-receive antenna ratio, $\alpha = M/N$		
constellation	1/2	1	2
QPSK	−15 dB	−17 dB	−21 dB
8-PSK	−18 dB	−20 dB	−26 dB
16-QAM	−21 dB	−23 dB	−31 dB
64-QAM*		−29 dB	

*The target for 64-QAM with symmetric antenna configuration is concluded from the numerical results of the journal version of this paper [2] for comparison.

A simple linear approximation that provides a lower bound for the EVM in the case of Gaussian signaling and $\alpha = 1$ reads [2]

$$\mathsf{EVM} = -0.7\cdot\bar{\gamma} - 13,$$

where $\bar{\gamma}$ is the SNR without transmit-side noise in decibels.

[1] E. Björnson, et al., "Capacity limits and multiplexing gains of MIMO channels with transceiver impairments," *IEEE CL*, (17)1, pp. 91–94, Jan 2013.

[2] M. Vehkaperä et al., "Asymptotic analysis of SU-MIMO channels with transmitter noise and mismatched joint decoding," *IEEE TCOM*, (63)3, pp. 749–765, Mar 2015.