**Summary**

Hardware non-idealities in wireless transmitter electronics cause distortion that is not captured by conventional linear channel models. Motivated by cross-sector magnitude (EVM) measurements in conformance testing, herein the achievable rate of a ‘homoisy’ multiple-input multiple-output (MIMO) channel

\[ y = H(x + v) + w \in \mathbb{C}^M, \]

is considered. The non-idealities manifest themselves as an additive noise term \( v \in \mathbb{C}^M \) at the transmit side. Large system analysis covering both Gaussian and practical digital modulation schemes is presented and numerical results illustrate how tolerable EVM levels depend non-trivially on various factors, such as: signal-to-noise ratio, modulation order and the level of asymmetry in antenna array configurations.

**System Model**

The system model related to the received signal given by (1) is depicted in the figure above. Transmitter is assumed to use spatial multiplexing and the receiver knows the PDFs of the noise plus distortion terms \( v \) and \( w \) as well as the distribution of the data vector \( x \). The conditional PDF

\[ f(y | x; H) = f(y | H) \cdot f(x; H) \]

is used for matched joint decoding of the transmitted signals.

**Achievable Rate**

In the ideal case, codewords span infinitely many independent channel realizations and the achievable rate is given by the ergodic mutual information (in nats)

\[ I(y; x) = \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{S}{N} \right), \]

where the outer expectations are w.r.t. all random variables in (1). The achievable rate \( I(y; x) \) was investigated in the case of Gaussian signaling in [1]. However, the case of practical digital modulation such as PSK and QAM has remained an open problem. Main goal of the paper: Evaluate (3) for PSK and QAM channel inputs.

**GOAL: Evaluate the Achievable Rate in (3)**

1. From (2) we get \( I(y; x) = \mathbb{E}_H \left( \log_2 \left( 1 + \frac{S}{N} \right) \right) \). Expectation over channel can be evaluated, e.g., using MC methods or random matrix theory (RMT). \( \mathbb{E}_H(h(y; x)) \) is “easy” to compute.

2. To obtain an expression for \( h(y; x) \), we need to evaluate a term:

\[ \sum_{x \in A} \mathbb{E}_H \left( \log_2 \left( 1 + \frac{S}{N} \right) \right), \]

where \( A \) is the modulation set (for example, PSK or QAM).

- \( S/N \) computation has exponential complexity and RMT does not work.
- The term \( h(y; x) \) seems intractable for conventional formulas.

Solution: Use the (non-rigorous) replica method from statistical physics.

**Result**

Consider the simplified case \( R_c = r_c J \) and \( R_n = r_n J \), i.e., the antennas experience spatially white distortion at both ends of the link. Then, for large \( M \), the ergodic MI of the original MIMO system (3) can be approximated as

\[ I(y; x) = \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{S}{N} \right), \]

where \( \mu, \nu \) (similarly \( \mu', \nu' \)) are solutions to coupled fixed point equations and \( I(y; x) \) is the MI of a scalar AWGN channel (see [2] and the paper for extensions and details).

Conclusion: The analysis of a fading MIMO channel reduces to analysis of an equivalent non-fading SISO channel!

\[ I(y; x) = \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{S}{N} \right), \]

**Numerical Examples**

- **Rate loss vs. SNR for antenna ratios \( \alpha = M/N \).**
- **SNR vs. transmit-to-receive antenna ratio \( \alpha \).**

**EVM Target for PSK and QAM**

If the EVM remains below the values tabulated in the table, hardware non-idealities can be considered negligible.

**Figure:** EVM values for PSK and QAM in the case of Gaussian signaling and \( \alpha = 1 \). The regions above curves define \( \gamma \) EVM pairs for rate losses higher than 5%, and vice versa for the areas below the curves.

\[ \text{Normalized rate } M^{-1} I(y; x) \text{ with ideal (EVM } = \infty \text{ dB) and non-ideal (EVM } = -10 \text{ dB) hardware. Markers for MC simulations with } M = N = 4. \]

\[ \text{Normalized rate } M^{-1} I(y; x) \text{ with ideal (EVM } = \infty \text{ dB) and non-ideal (EVM } = -10 \text{ dB) hardware. Markers for MC simulations with } M = N = 4. \]

**References**
