**Aalto University**
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Engineering

# Power Control and Beamformer Design for the Optimization of Full-Duplex MIMO Relays in a Dual-Hop MISO Link

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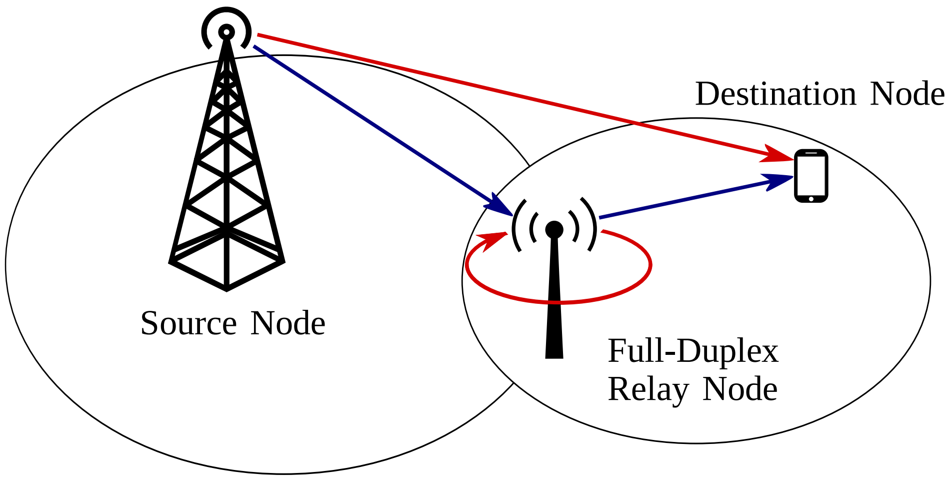
# Introduction

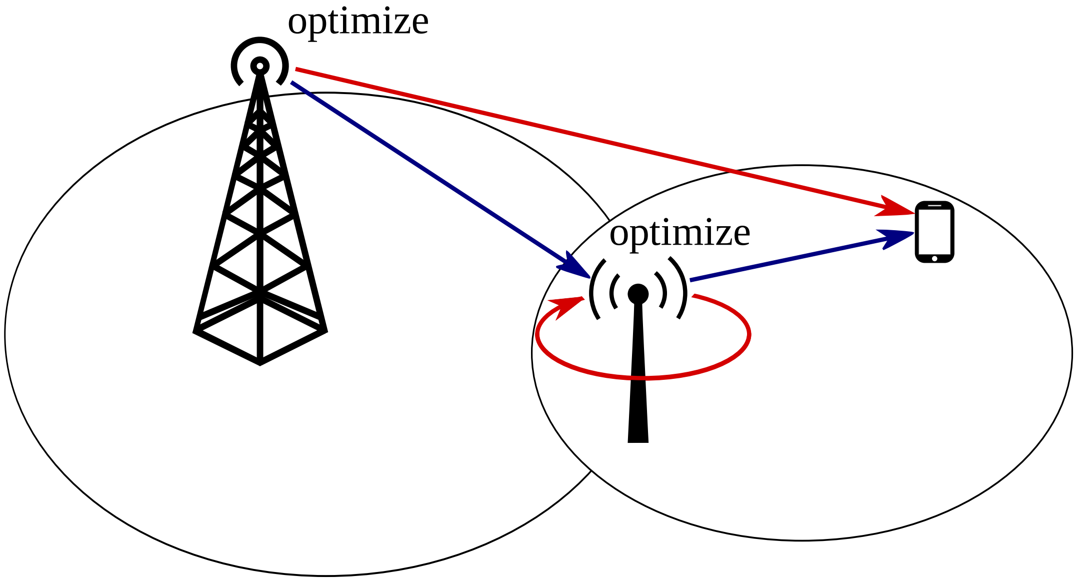
# Full-Duplex MIMO Relay in Dual-Hop MISO Link



- Single-stream end-to-end full-duplex transmission
  - Source: $M_{\mathrm{S}}$ antennas
  - Relay with self-interference:
    $N_r$ receive antennas
    $N_t$ transmit antennas
  - Destination: one antenna

- Both common relaying strategies are considered in this work:
  - amplify-and-forward (AF)
  - decode-and-forward (DF)
- The direct source–destination link is assumed to be weak and included as interference (since otherwise switching to a direct transmission mode may be preferred)

# Power Control and Beamformer Design

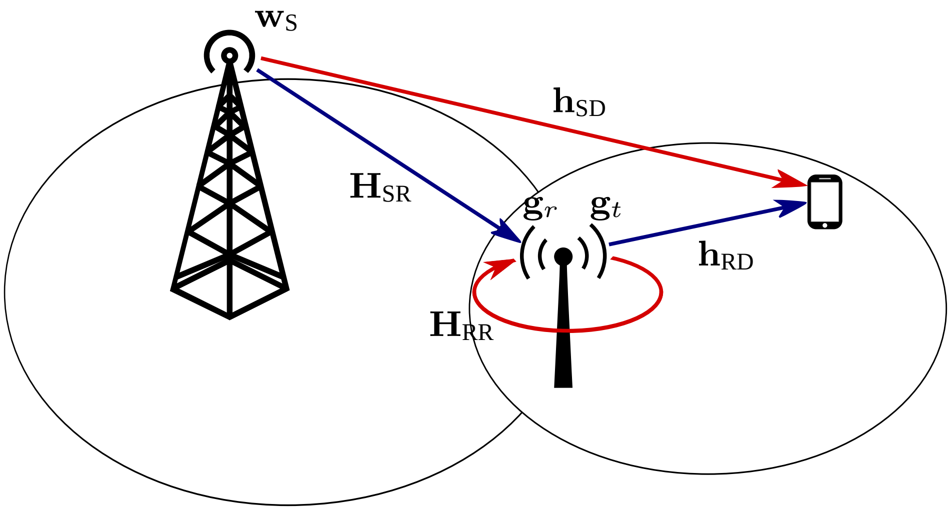


- The general objective: $$\boxed{\max_{p_\mathrm{S}, p_\mathrm{R}, \mathbf{w}_\mathrm{S}, \mathbf{g}_r, \mathbf{g}_t} \log_2\left(1 + \gamma_\mathrm{e2e}\right)}$$

  subject to $0 < p_\mathrm{S} \leq P_\mathrm{S}$, $0 \leq p_\mathrm{R} \leq P_\mathrm{R}$, and $\|\mathbf{w}_\mathrm{S}\| = \|\mathbf{g}_r\| = \|\mathbf{g}_t\| = 1$
  - ▷ Source: Tx power adaptation and Tx beamforming
  - ▷ Relay: Tx power adaptation and joint Rx–Tx beamforming

# System Model

# End-to-End Signal Model

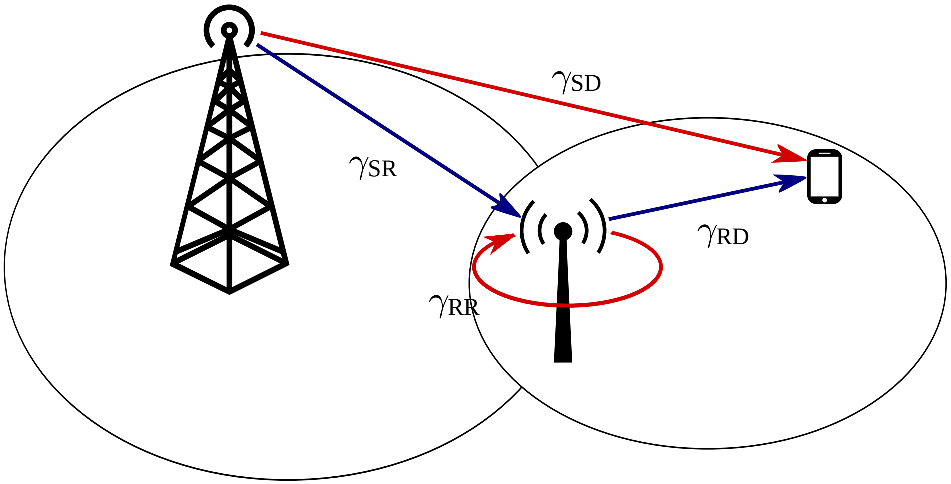


- Received signals at the relay and at the destination:

$$\begin{aligned} y_\mathrm{R} &= \mathbf{g}_r \mathbf{H}_\mathrm{SR} \mathbf{w}_\mathrm{S} x_\mathrm{S} + \mathbf{g}_r \mathbf{H}_\mathrm{RR} \mathbf{g}_t \widetilde{x}_\mathrm{R} + \mathbf{g}_r \mathbf{n}_\mathrm{R} \\ y_\mathrm{D} &= \mathbf{h}_\mathrm{RD} \mathbf{g}_t x_\mathrm{R} + \mathbf{h}_\mathrm{SD} \mathbf{w}_\mathrm{S} \widetilde{x}_\mathrm{S} + n_\mathrm{D} \end{aligned}$$

- Due to processing delay, $\mathbb{E}\{x_\mathrm{S} x_\mathrm{R}\} = \mathbb{E}\{x_\mathrm{S} \widetilde{x}_\mathrm{S}\} = \mathbb{E}\{x_\mathrm{R} \widetilde{x}_\mathrm{R}\} = 0$

# Signal-to-[Interference-and-]Noise Ratios (S[I]NRs)



- Channel SNRs (post-processing):

$$\gamma_{\mathrm{SR}} = \frac{|\mathbf{g}_r \mathbf{H}_{\mathrm{SR}} \mathbf{w}_{\mathrm{S}}|^2}{\sigma_{\mathrm{R}}^2},\ \gamma_{\mathrm{RD}} = \frac{|\mathbf{h}_{\mathrm{RD}} \mathbf{g}_t|^2}{\sigma_{\mathrm{D}}^2},\ \gamma_{\mathrm{RR}} = \frac{|\mathbf{g}_r \mathbf{H}_{\mathrm{RR}} \mathbf{g}_t|^2}{\sigma_{\mathrm{R}}^2},\ \gamma_{\mathrm{SD}} = \frac{|\mathbf{h}_{\mathrm{SD}} \mathbf{w}_{\mathrm{S}}|^2}{\sigma_{\mathrm{D}}^2}$$

- End-to-end SINRs [expressions (7) and (8) in the paper]:

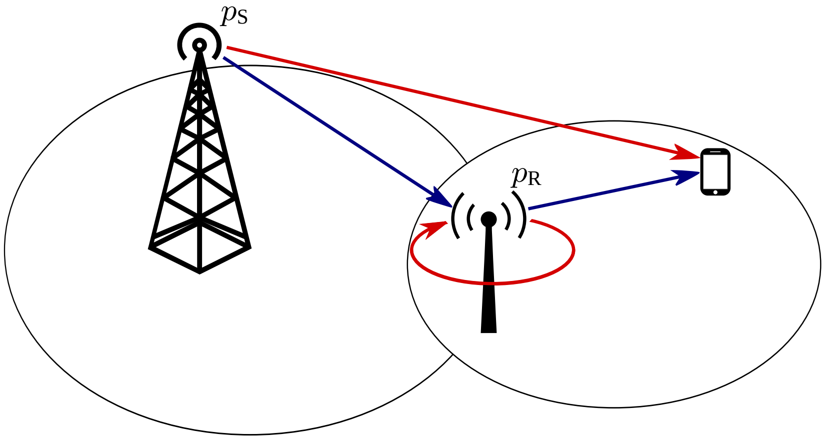
$$\gamma_{\mathrm{e2e}}^{\mathrm{AF}} = \frac{\gamma_{\mathrm{R}}\gamma_{\mathrm{D}}}{\gamma_{\mathrm{R}}+\gamma_{\mathrm{D}}+1} \quad \text{and} \quad \gamma_{\mathrm{e2e}}^{\mathrm{DF}} = \min\{\gamma_{\mathrm{R}}, \gamma_{\mathrm{D}}\}$$

where link SINRs are given by $\gamma_{\mathrm{R}} = \frac{p_{\mathrm{S}}\gamma_{\mathrm{SR}}}{p_{\mathrm{R}}\gamma_{\mathrm{RR}}+1}$ and $\gamma_{\mathrm{D}} = \frac{p_{\mathrm{R}}\gamma_{\mathrm{RD}}}{p_{\mathrm{S}}\gamma_{\mathrm{SD}}+1}$

# Power Control and Beamformer Design

# Power Control with Fixed Beamformers

- Consider fixed beamformers ($\|\mathbf{w}_\mathrm{S}\| = \|\mathbf{g}_r\| = \|\mathbf{g}_t\| = 1$)



- Either transmit power, when fixing the other power level, can be optimized explicitly with both relaying strategies:

  $\hat{p}_\mathrm{S}(p_\mathrm{R})$ and $\hat{p}_\mathrm{R}(p_\mathrm{S})$ [expressions (9), (10), (12) and (13) in the paper]

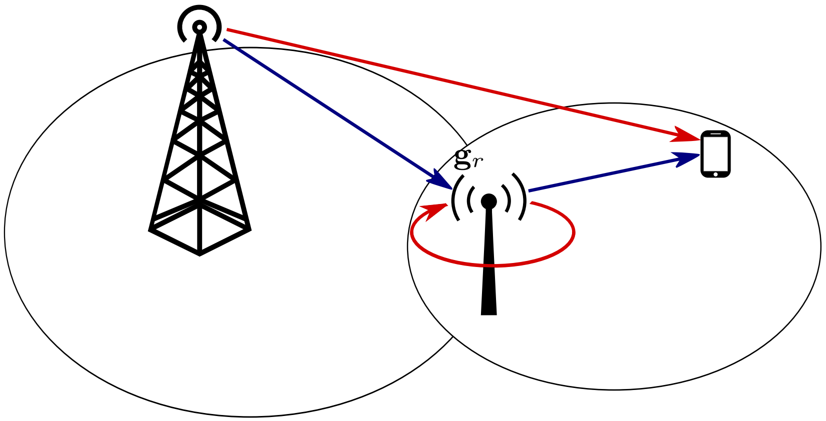
  - ▷ Clipping: $[p_\mathrm{S}, p_\mathrm{R}] = \begin{cases} [P_\mathrm{S}, \hat{p}_\mathrm{R}(P_\mathrm{S})], & \gamma_\mathrm{e2e}\big|_{p_\mathrm{R}=P_\mathrm{R}} \leq \gamma_\mathrm{e2e}\big|_{p_\mathrm{S}=P_\mathrm{S}} \\ [\hat{p}_\mathrm{S}(P_\mathrm{R}), P_\mathrm{R}], & \text{otherwise} \end{cases}$

# Beamformer Design by Minimum MSE

- Consider fixed Tx beamformers ($\|\mathbf{w}_{\mathrm{S}}\| = \|\mathbf{g}_t\| = 1$)
- Optimal MMSE filtering:

$$\mathbf{g}_r(p_{\mathrm{R}}) = \frac{\mathbf{w}_{\mathrm{S}}^H \mathbf{H}_{\mathrm{SR}}^H \mathbf{R}_{\mathrm{I+N}}^{-1}}{\|\mathbf{w}_{\mathrm{S}}^H \mathbf{H}_{\mathrm{SR}}^H \mathbf{R}_{\mathrm{I+N}}^{-1}\|}$$

where $\mathbf{R}_{\mathrm{I+N}} = p_{\mathrm{R}} \mathbf{H}_{\mathrm{RR}} \mathbf{g}_t \mathbf{g}_t^H \mathbf{H}_{\mathrm{RR}}^H + \sigma_{\mathrm{R}}^2 \mathbf{I}$



- Either transmit power, when fixing the other power level, can be again optimized explicitly with both relaying strategies:

$\hat{p}_{\mathrm{S}}(p_{\mathrm{R}})$ and $\hat{p}_{\mathrm{R}}(p_{\mathrm{S}})$ [expressions (16)–(28) and (29)–(36) in the paper]

▷ Clipping: $$[p_{\mathrm{S}}, p_{\mathrm{R}}] = \begin{cases} [P_{\mathrm{S}}, \hat{p}_{\mathrm{R}}(P_{\mathrm{S}})], & \gamma_{\mathrm{e2e}}\big|_{p_{\mathrm{R}}=P_{\mathrm{R}}} \leq \gamma_{\mathrm{e2e}}\big|_{p_{\mathrm{S}}=P_{\mathrm{S}}} \\ [\hat{p}_{\mathrm{S}}(P_{\mathrm{R}}), P_{\mathrm{R}}], & \text{otherwise} \end{cases}$$

# Beamformer Design by Maximum SLNR

- The leakage signal is defined as the aggregate transmitted signal which is received and interpreted as interference by others
- Signal-to-leakage-plus-noise ratios (SLNRs):

$$\widetilde{\gamma}_{\mathrm{R}} = \frac{|\mathbf{g}_r \mathbf{H}_{\mathrm{SR}} \mathbf{w}_{\mathrm{S}}|^2 p_{\mathrm{S}}}{|\mathbf{h}_{\mathrm{SD}} \mathbf{w}_{\mathrm{S}}|^2 p_{\mathrm{S}} + \sigma_{\mathrm{R}}^2} \quad \text{and} \quad \widetilde{\gamma}_{\mathrm{D}} = \frac{|\mathbf{h}_{\mathrm{RD}} \mathbf{g}_t|^2 p_{\mathrm{R}}}{|\mathbf{g}_r \mathbf{H}_{\mathrm{RR}} \mathbf{g}_t|^2 p_{\mathrm{R}} + \sigma_{\mathrm{D}}^2}$$

  - ▷ The optimization of transmit beamformers is decoupled:

$$\arg\max_{\mathbf{w}_{\mathrm{S}}} \widetilde{\gamma}_{\mathrm{R}} = \left(p_{\mathrm{S}} \mathbf{h}_{\mathrm{SD}}^H \mathbf{h}_{\mathrm{SD}} + \sigma_{\mathrm{R}}^2 \mathbf{I}\right)^{-1} \mathbf{H}_{\mathrm{SR}}^H \mathbf{g}_r^H$$

$$\arg\max_{\mathbf{g}_t} \widetilde{\gamma}_{\mathrm{D}} = \left(p_{\mathrm{R}} \mathbf{H}_{\mathrm{RR}}^H \mathbf{g}_r^H \mathbf{g}_r \mathbf{H}_{\mathrm{RR}} + \sigma_{\mathrm{D}}^2 \mathbf{I}\right)^{-1} \mathbf{h}_{\mathrm{RD}}^H$$

- The SLNR-based transmit beamformers are optimal if $p_{\mathrm{S}}\gamma_{\mathrm{SR}} = p_{\mathrm{R}}\gamma_{\mathrm{RD}}$ or $p_{\mathrm{S}}\gamma_{\mathrm{SD}} = p_{\mathrm{R}}\gamma_{\mathrm{RR}}$ assuming $\sigma_{\mathrm{R}}^2 = \sigma_{\mathrm{D}}^2$

# Beamformer Design by Joint Minimum MSE and Maximum SLNR

- A set of joint iterations between the min-MSE receive beamformer with optimal transmit powers and max-SLNR transmit beamformers
- Convergence is not guaranteed due to the nonconvexity of the transmit power expressions
  - ▷ The algorithm can be terminated when the end-to-end SINR does not increase anymore

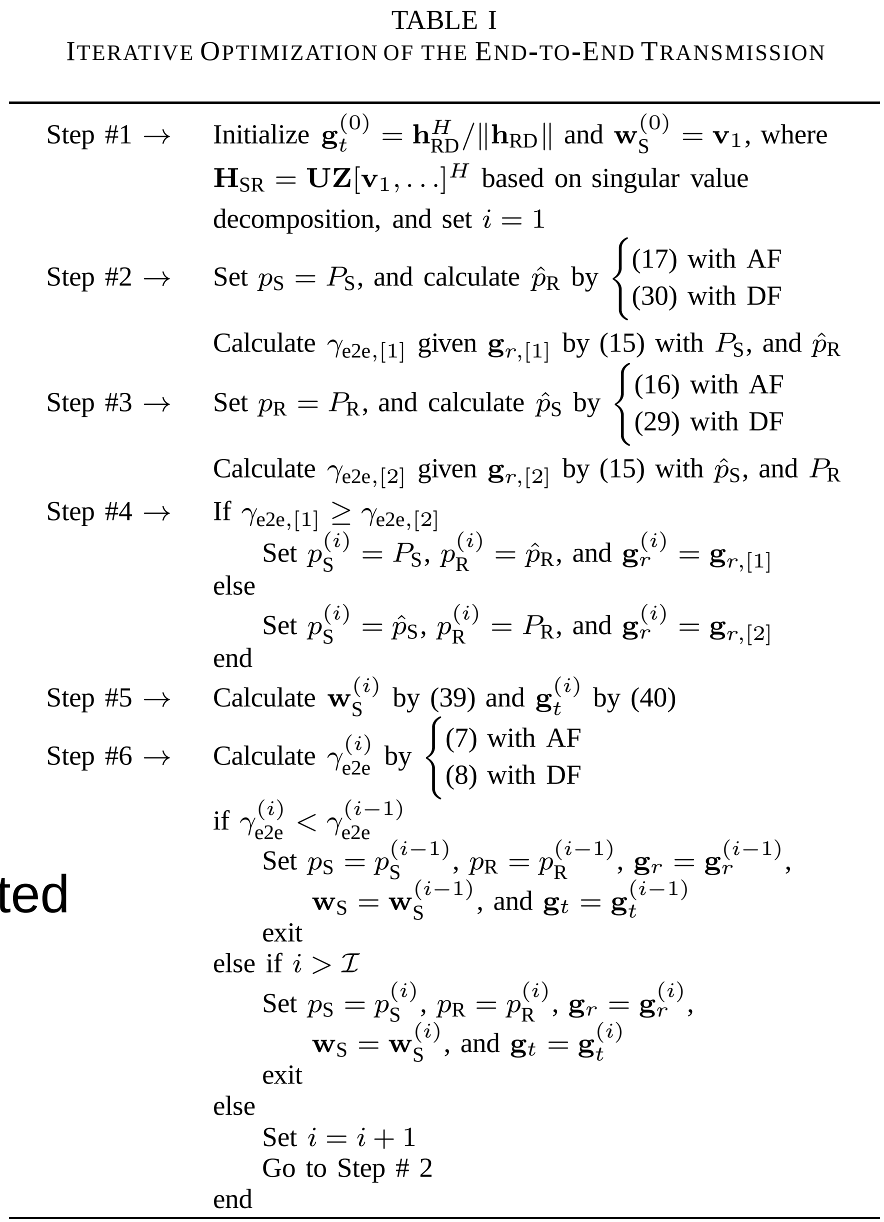


TABLE I
ITERATIVE OPTIMIZATION OF THE END-TO-END TRANSMISSION

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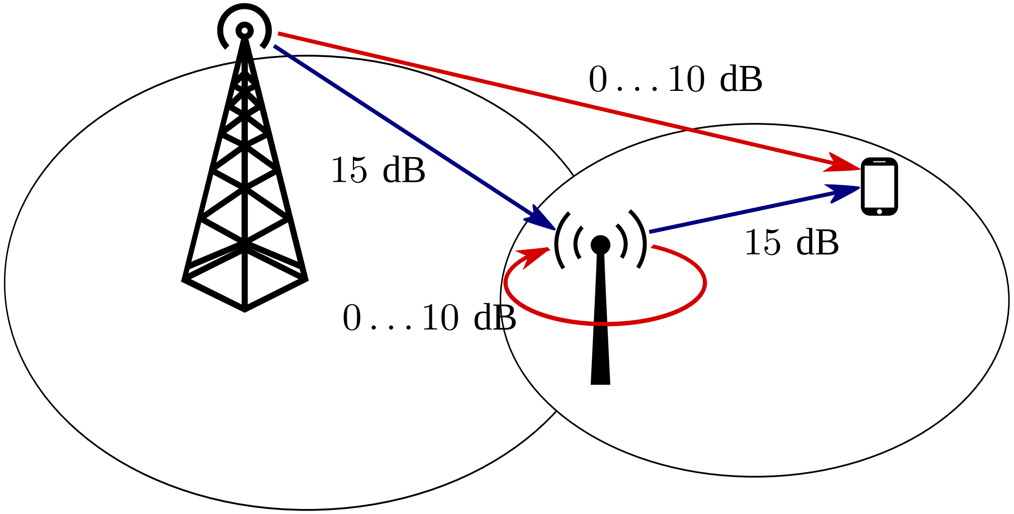
Step #1 →  Initialize $\mathbf{g}_t^{(0)} = \mathbf{h}_{\text{RD}}^H/\|\mathbf{h}_{\text{RD}}\|$ and $\mathbf{w}_{\text{S}}^{(0)} = \mathbf{v}_1$, where
           $\mathbf{H}_{\text{SR}} = \mathbf{U}\mathbf{Z}[\mathbf{v}_1, \ldots]^H$ based on singular value
           decomposition, and set $i = 1$
Step #2 →  Set $p_{\text{S}} = P_{\text{S}}$, and calculate $\hat{p}_{\text{R}}$ by $\begin{cases}(17) \text{ with AF}\\ (30) \text{ with DF}\end{cases}$
           Calculate $\gamma_{\text{e2e},[1]}$ given $\mathbf{g}_{r,[1]}$ by (15) with $P_{\text{S}}$, and $\hat{p}_{\text{R}}$
Step #3 →  Set $p_{\text{R}} = P_{\text{R}}$, and calculate $\hat{p}_{\text{S}}$ by $\begin{cases}(16) \text{ with AF}\\ (29) \text{ with DF}\end{cases}$
           Calculate $\gamma_{\text{e2e},[2]}$ given $\mathbf{g}_{r,[2]}$ by (15) with $\hat{p}_{\text{S}}$, and $P_{\text{R}}$
Step #4 →  If $\gamma_{\text{e2e},[1]} \geq \gamma_{\text{e2e},[2]}$
               Set $p_{\text{S}}^{(i)} = P_{\text{S}}$, $p_{\text{R}}^{(i)} = \hat{p}_{\text{R}}$, and $\mathbf{g}_r^{(i)} = \mathbf{g}_{r,[1]}$
           else
               Set $p_{\text{S}}^{(i)} = \hat{p}_{\text{S}}$, $p_{\text{R}}^{(i)} = P_{\text{R}}$, and $\mathbf{g}_r^{(i)} = \mathbf{g}_{r,[2]}$
           end
Step #5 →  Calculate $\mathbf{w}_{\text{S}}^{(i)}$ by (39) and $\mathbf{g}_t^{(i)}$ by (40)
Step #6 →  Calculate $\gamma_{\text{e2e}}^{(i)}$ by $\begin{cases}(7) \text{ with AF}\\ (8) \text{ with DF}\end{cases}$
           if $\gamma_{\text{e2e}}^{(i)} < \gamma_{\text{e2e}}^{(i-1)}$
               Set $p_{\text{S}} = p_{\text{S}}^{(i-1)}$, $p_{\text{R}} = p_{\text{R}}^{(i-1)}$, $\mathbf{g}_r = \mathbf{g}_r^{(i-1)}$,
                   $\mathbf{w}_{\text{S}} = \mathbf{w}_{\text{S}}^{(i-1)}$, and $\mathbf{g}_t = \mathbf{g}_t^{(i-1)}$
               exit
           else if $i > \mathcal{I}$
               Set $p_{\text{S}} = p_{\text{S}}^{(i)}$, $p_{\text{R}} = p_{\text{R}}^{(i)}$, $\mathbf{g}_r = \mathbf{g}_r^{(i)}$,
                   $\mathbf{w}_{\text{S}} = \mathbf{w}_{\text{S}}^{(i)}$, and $\mathbf{g}_t = \mathbf{g}_t^{(i)}$
               exit
           else
               Set $i = i + 1$
               Go to Step # 2
           end
```

# Numerical Analysis

# Simulation Setup

- Example antenna configuration:
  - $M_{\mathrm{S}} = 4$
  - $N_r = 2$
  - $N_t = 2$



- Frequency-flat Rayleigh fading channels
  - constant example gain levels for the source–relay and relay–destination channels
  - ramping up the interference levels

# Joint Iterative Scheme vs. Conventional Design

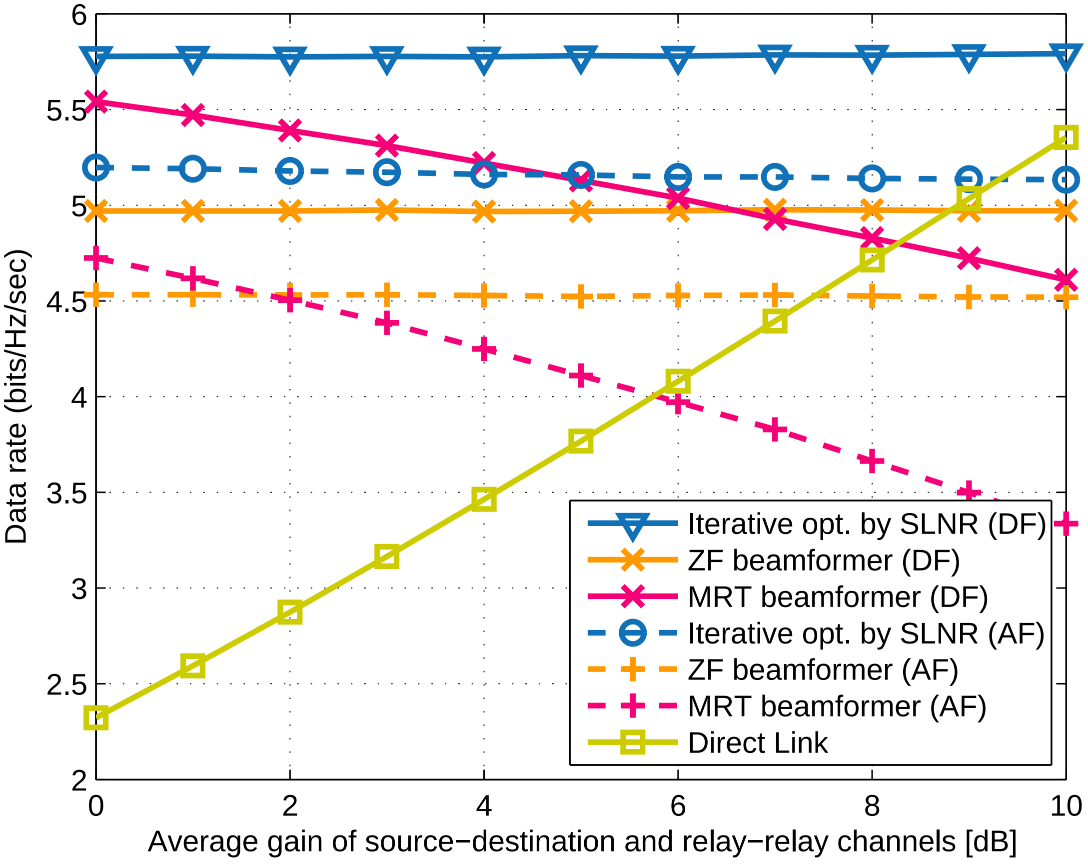


Fig. 2. The achievable data rates by the joint iterative algorithm vs. the conventional transmit beamformers.

# Gain from SLNR-based Transmit Beamformers

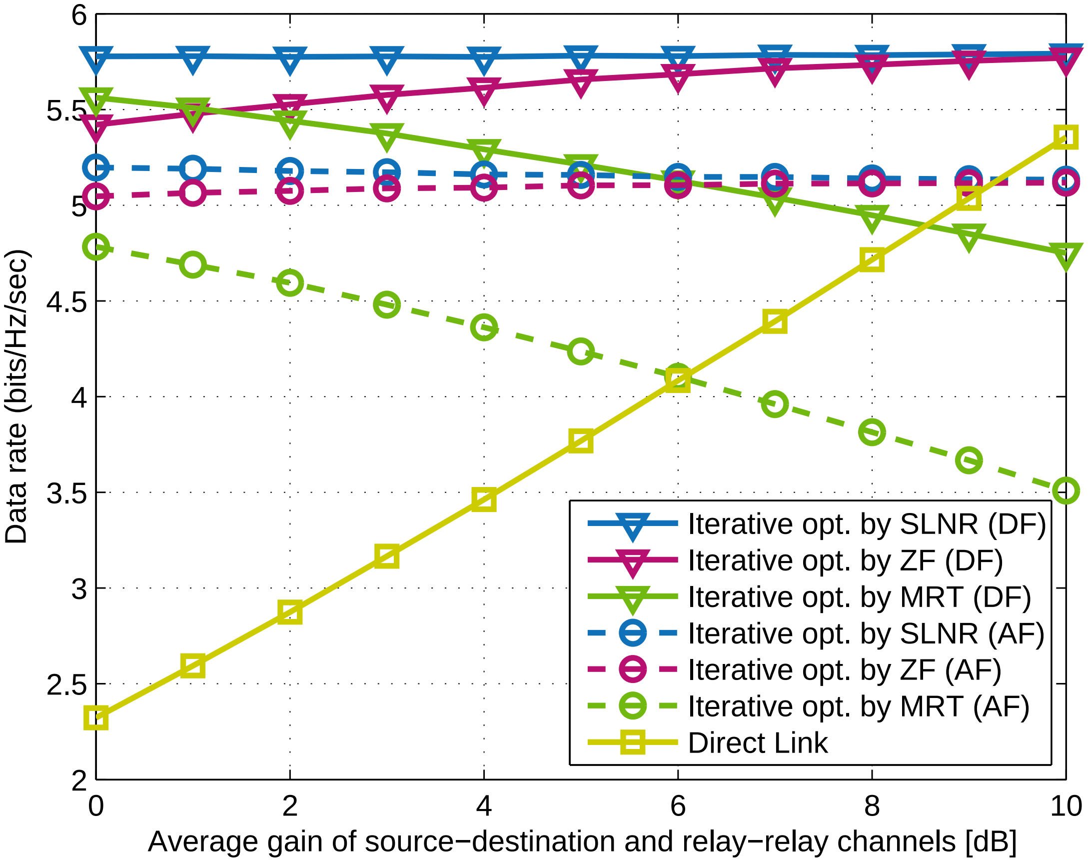


Fig. 3. The achievable data rates by the joint iterative algorithm with SLNR, MRT and ZF-based transmit beamformers.

# Conclusion

# Conclusion

- Scope: The optimization of a dual-hop MISO link with a full-duplex MIMO relay
    - ▷ either an amplify-and-forward or a decode-and-forward relay
    - ▷ the direct link interpreted as interference at the destination
- Design of transmit power control and beamforming filters to optimize the end-to-end transmission
    1. the optimal MMSE-based receive beamformer combined with the optimal transmit powers to tackle the non-convexity problem inherent to an iterative approach
    2. signal-to-leakage-plus-noise ratio as a figure of merit to approximate the optimal transmit beamformers
    3. an iterative algorithm to jointly optimize the receive and transmit beamformers

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