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Asymptotic Analysis of Full-Duplex Bidirectional MIMO Link with Transmitter Noise

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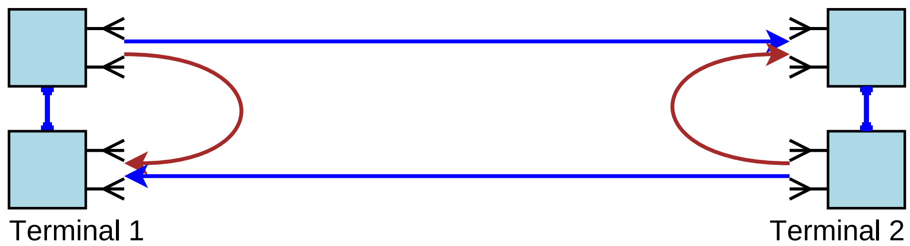
Introduction

Full-Duplex Wireless Communication Systems

- Advanced radio transceivers which are able to simultaneously transmit and receive (STAR) *on a single frequency band*
  - Potential applications of full-duplex operation are many
  - This work concentrates on bidirectional links
- Full duplex renders up to double spectral efficiency at link level!
  ...but there is still a significant technical challenge:
  the mitigation of unavoidable *self-interference*
  - Two main digital techniques:
    - *time-domain cancellation*
    - *spatial-domain suppression*

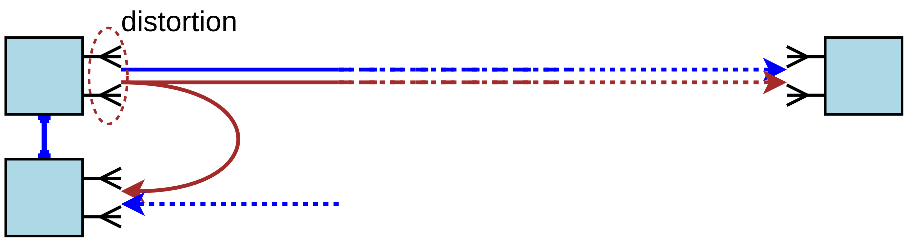
Asymptotic Analysis of Bidirectional Links



- Bidirectional *full-duplex* multiantenna (MIMO) link
  - ▷ large number of antennas
  - ▷ symmetric configuration in numerical results

- *Achievable rates*, i.e., (generalized) mutual information
- *The replica method*, originally developed in the field of statistical physics and recently applied to communication theory problems
  - ▷ necessitates the assumption of the *large-system limit* where the degrees of freedom in the system grow without bound
  - ▷ However, simulated (Monte Carlo) results for small-scale systems agree well with the corresponding asymptotic results

Transmitter Noise and M(ism)atched Decoding

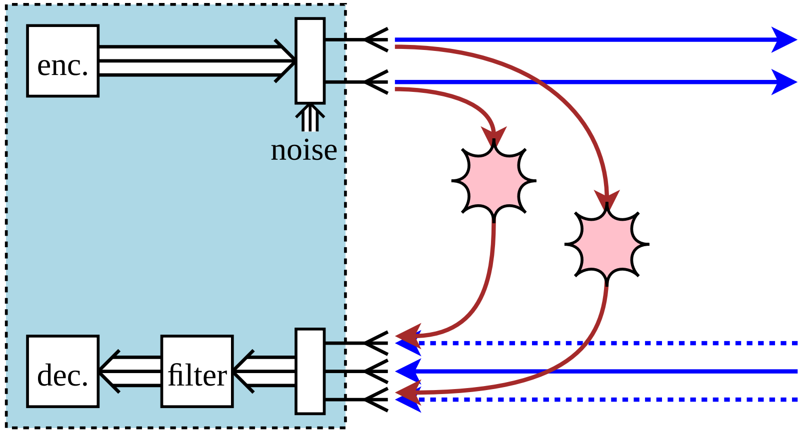


- *Unknown* transmit-side noise due to analog imperfections
  - ▷ nonlinear distortion, e.g., power amplifier (PA)
  - ▷ measured with EVM

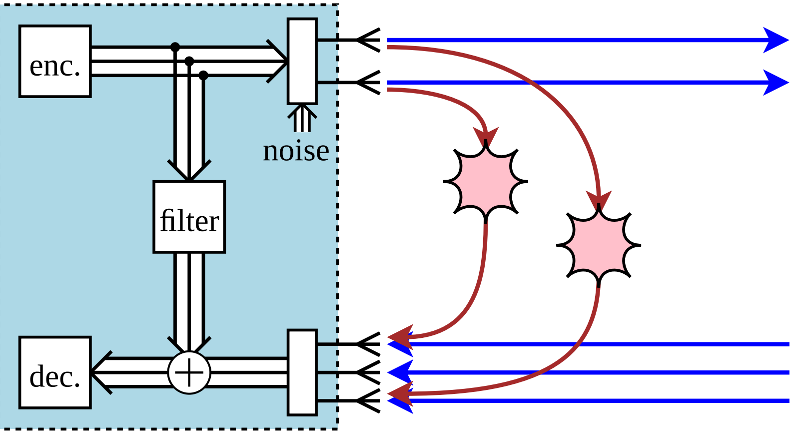
- *Feedback* transmit-side noise may be on a par with the far-end signal due to the high gain of the near-end interference channel
  - ▷ *Feedforward* transmit-side noise can be neglected since it is typically below receive-side noise after channel attenuation
- Mitigation transparently around the actual multiplexing protocol which can operate without being aware of self-interference
  - ▷ *Mismatched* detection and decoding due to unexpected noise

Self-interference Mitigation with Transmitter Noise

Spatial-domain suppression:



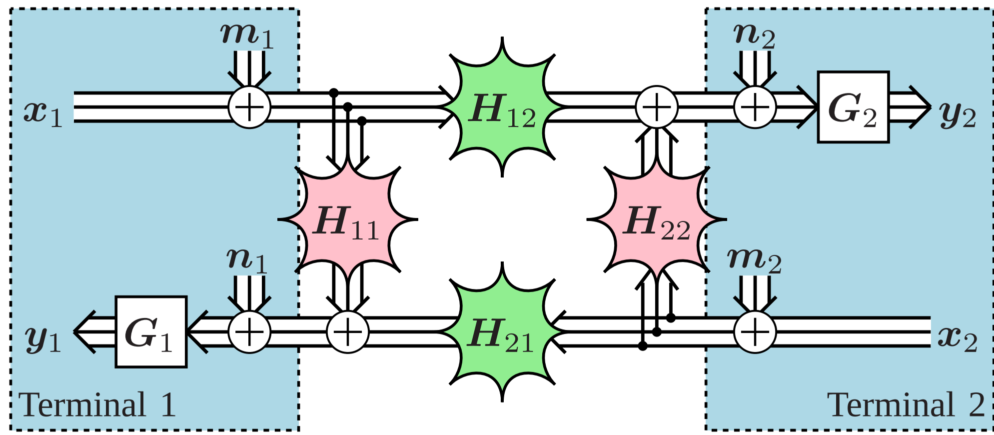
Time-domain cancellation:



- The link needs efficient self-interference mitigation at both ends
  - **Suppression**: receiving only in the null space of interference
  - **Cancellation**: subtracting the interfering signal before decoder
- Both can eliminate the data-dependent part of self-interference
- Suppression eliminates also the self-induced transmit-side noise, at the cost of consuming some spatial degrees of freedom

System Model

Signal Model (Fig. 1)



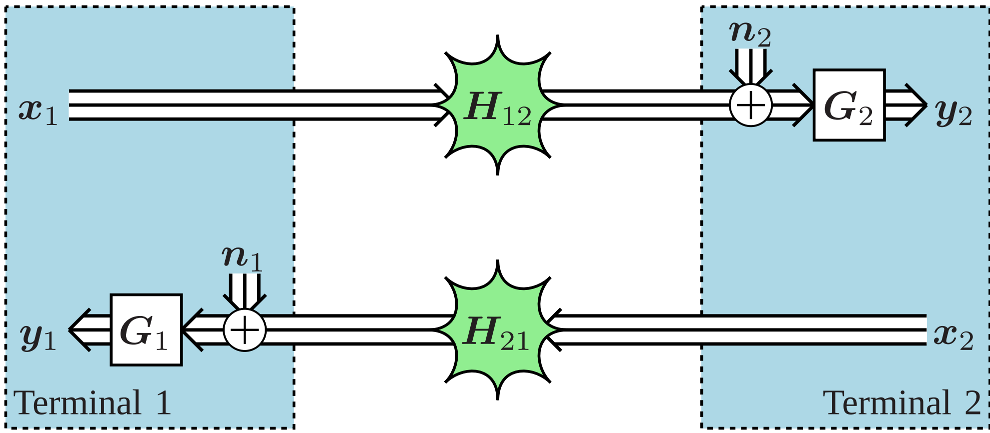
- Terminal $i \in \{1, 2\}$ has M_i transmit and N_i receive antennas
- In communication direction $ij \in \{12, 21\}$:

$$\boldsymbol{y}_j = \boldsymbol{G}_j \boldsymbol{H}_{ij}(\boldsymbol{x}_i + \boldsymbol{m}_i) + \boldsymbol{G}_j \boldsymbol{H}_{jj}(\boldsymbol{x}_j + \boldsymbol{m}_j) + \boldsymbol{G}_j \boldsymbol{n}_j$$

  - ▷ noise terms $\boldsymbol{m}_i$ and $\boldsymbol{m}_j$ due to transmitter imperfections
  - ▷ $\hat{N}_j$ receive streams remain after self-interference mitigation

- Terminal i does not know $\boldsymbol{H}_{ij}$ but Terminal j knows $\boldsymbol{H}_{ij}$ and $\boldsymbol{H}_{jj}$

Spatial-Domain Suppression

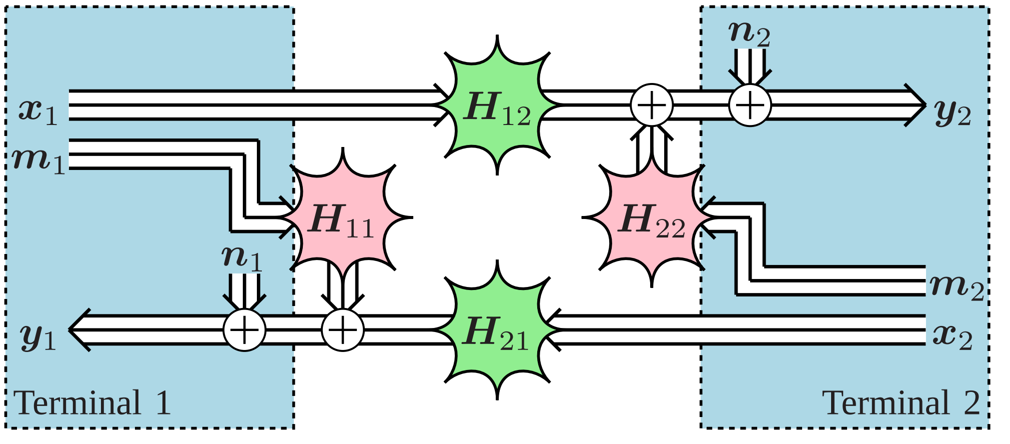


- In Terminal $j \in \{1, 2\}$ after suppression using $\boldsymbol{G}_j$ of rank $\hat{N}_j$:

$$\boldsymbol{y}_j = \boldsymbol{G}_j(\boldsymbol{H}_{ij}\boldsymbol{x}_i + \underbrace{\boldsymbol{H}_{ij}\boldsymbol{m}_i}_{\approx \boldsymbol{0}}) + \underbrace{\boldsymbol{G}_j\boldsymbol{H}_{jj}(\boldsymbol{x}_j + \boldsymbol{m}_j)}_{\text{eliminated when } \boldsymbol{G}_j\boldsymbol{H}_{jj}=\boldsymbol{0}} + \boldsymbol{G}_j\boldsymbol{n}_j$$

- $\hat{N}_j = N_j - M_j$ if $\boldsymbol{H}_{jj}$ has full rank, thus requiring $N_j > M_j$
- When enclosing any conventional (e.g., half-duplex) transceiver by transparent suppression, it still performs *matched decoding*

Time-Domain Cancellation



- In Terminal $j \in \{1, 2\}$ after cancellation presuming $\boldsymbol{G}_j = \boldsymbol{I}$:

$$\boldsymbol{y}_j = \boldsymbol{H}_{ij}\boldsymbol{x}_i + \underbrace{\boldsymbol{H}_{ij}\boldsymbol{m}_i}_{\approx \boldsymbol{0}} + \underbrace{\boldsymbol{H}_{jj}\boldsymbol{x}_j}_{\text{eliminated}} + \boldsymbol{H}_{jj}\underbrace{\boldsymbol{m}_j}_{\text{unknown!}} + \boldsymbol{n}_j$$

- $\hat{N}_j = N_j$, i.e., all degrees of freedom are saved for data reception
- Conventional receivers may adapt imperfectly to the presence of unexpected transmitter noise, leading to *mismatched decoding*

Analytical Results

Problem Statement

- "Unified" signal model: $\boldsymbol{y}_j \simeq \boldsymbol{G}_j \boldsymbol{H}_{ij} \boldsymbol{x}_i + \boldsymbol{w}_j$

  where $\boldsymbol{w}_j = \boldsymbol{G}_j \boldsymbol{H}_{jj} \boldsymbol{m}_j + \boldsymbol{G}_j \boldsymbol{n}_j$ with $\boldsymbol{R}_{\boldsymbol{w}_j} = \frac{\sigma_j^2}{M_j} \boldsymbol{G}_j \boldsymbol{H}_{jj} \boldsymbol{H}_{jj}^H \boldsymbol{G}_j^H + \boldsymbol{I}$

  1. Matched decoding uses the true density $p(\boldsymbol{y}_j \mid \boldsymbol{x}_i, \mathcal{H}_{ij})$
  2. Mismatched decoding estimates $\boldsymbol{R}_{\boldsymbol{w}_j}$ as $\tilde{\boldsymbol{R}}_{\boldsymbol{w}_j}$ and uses a postulated density $q(\boldsymbol{y}_j \mid \boldsymbol{x}_i, \mathcal{H}_{ij})$

- Generalized mutual information (GMI) is defined as

$$\boxed{I_{\mathsf{gmi}}(\boldsymbol{y}_j; \boldsymbol{x}_i) = \sup_{s>0} I_{\mathsf{gmi}}^{(s)}(\boldsymbol{y}_j; \boldsymbol{x}_i) = \sup_{s>0} \Big(\mathsf{E} \ln q(\boldsymbol{y}_j \mid \boldsymbol{x}_i, \mathcal{H}_{ij})^s - \mathsf{E} \ln q^{(s)}(\boldsymbol{y}_j \mid \mathcal{H}_{ij}) \Big)}$$

  where $q^{(s)}(\boldsymbol{y}_j \mid \mathcal{H}_{ij}) = \mathsf{E}_{\boldsymbol{x}_i} q(\boldsymbol{y}_j \mid \boldsymbol{x}_i, \mathcal{H}_{ij})^s$

- The first term is easy to calculate, yielding

$$I_{\mathsf{gmi}}^{(s)}(\boldsymbol{y}_j; \boldsymbol{x}_i) = \Big(c - s \, \mathsf{E} \, \mathrm{tr}(\tilde{\boldsymbol{R}}_{\boldsymbol{w}_j}^{-1} \boldsymbol{R}_{\boldsymbol{w}_j}) \Big) - \mathsf{E} \ln q^{(s)}(\boldsymbol{y}_j \mid \mathcal{H}_{ij}),$$

  while the second term needs special tricks as follows

Replica Analysis

- Instead of trying direct calculation, let us take a different route and start by reformulating the difficult term as

$$\mathsf{E} \ln q^{(s)}(\boldsymbol{y}_j \mid \mathcal{H}_{ij}) = c + \lim_{u \to 0} \frac{\partial}{\partial u} \ln \mathsf{E} Z(\boldsymbol{y}_j, \mathcal{H}_{ij}; s)^u$$

  where $Z(\boldsymbol{y}_j, \mathcal{H}_{ij}; s) = \mathsf{E}_{\boldsymbol{x}_i} \mathrm{e}^{-(\boldsymbol{y}_j - \boldsymbol{G}_j \boldsymbol{H}_{ij} \boldsymbol{x}_i)^H s \tilde{\boldsymbol{R}}_{\boldsymbol{w}_j}^{-1} (\boldsymbol{y}_j - \boldsymbol{G}_j \boldsymbol{H}_{ij} \boldsymbol{x}_i)}$
- To circumvent the problem of u being real-valued, the replica trick then postulates

$$Z(\boldsymbol{x}_0, \boldsymbol{w}_j, \mathcal{H}_{ij}; s)^u = \mathsf{E}_{\{\boldsymbol{x}_a\}_{a=1}^u} \prod_{a=1}^{u} \mathrm{e}^{-[\boldsymbol{w}_j + \boldsymbol{G}_j \boldsymbol{H}_{ij} (\boldsymbol{x}_0 - \boldsymbol{x}_a)]^H s \tilde{\boldsymbol{R}}_{\boldsymbol{w}_j}^{-1} [\boldsymbol{w}_j + \boldsymbol{G}_j \boldsymbol{H}_{ij} (\boldsymbol{x}_0 - \boldsymbol{x}_a)]}$$

  where $\boldsymbol{x}_0$ and $\{\boldsymbol{x}_a\}_{a=1}^u$ denote the original and replicated vectors
- If we manage to assess the above expectation as a function of u when matrix dimensions in $\mathcal{H}_{ij}$ grow without bound with fixed ratios, analytically continuing $u \to 0$ recovers the per-stream GMI as

$$\frac{1}{M} I_{\mathsf{gmi}}^{(s)}(\boldsymbol{y}_j; \boldsymbol{x}_i) = -\frac{s}{M} \mathsf{E}\, \mathrm{tr}(\tilde{\boldsymbol{R}}_{\boldsymbol{w}_j}^{-1} \boldsymbol{R}_{\boldsymbol{w}_j}) - \lim_{M \to \infty} \frac{1}{M} \lim_{u \to 0} \frac{\partial}{\partial u} \ln \mathsf{E} Z(\boldsymbol{x}_0, \boldsymbol{w}_j, \mathcal{H}_{ij}; s)^u$$

Matched Decoding: Per-stream Achievable Rate

- When $\boldsymbol{H}_{ij}$ and $\boldsymbol{H}_{jj}$ are i.i.d. Gaussian with gains $\bar{\gamma}_{ij}$ and $\bar{\gamma}_{jj}$ and the receiver adapts perfectly to residual self-interference:

$$\frac{R_{ij}}{M_i} = \ln(1+\eta_{ij}) - \frac{\eta_{ij}}{1+\eta_{ij}} + \frac{1}{\alpha_{ij}}\left[I\left(\alpha_{jj}, \bar{\gamma}_{jj}\sigma_j^2;\, 1+\frac{\bar{\gamma}_{ij}}{1+\eta_{ij}}\right) - I(\alpha_{jj}, \bar{\gamma}_{jj}\sigma_j^2;\, 1)\right]$$

for which the fixed-point η_{ij} is found numerically by iterating

$$\eta_{ij} = \frac{\bar{\gamma}_{ij}}{\alpha_{ij}}\left[\frac{1}{1+\frac{\bar{\gamma}_{ij}}{1+\eta_{ij}}} - \frac{\alpha_{ii}}{4\bar{\gamma}_{jj}\sigma_j^2}\mathcal{F}\left(\frac{\bar{\gamma}_{jj}\sigma_j^2}{\alpha_{ii}} \cdot \frac{1}{1+\frac{\bar{\gamma}_{ij}}{1+\eta_{ij}}}, \alpha_{ii}\right)\right]$$

and the auxiliary functions are given by

$$\mathcal{F}(x,\beta) = \left(\sqrt{x(1+\sqrt{\beta})^2+1} - \sqrt{x(1-\sqrt{\beta})^2+1}\right)^2$$

$$I(\beta,\sigma^2;\, t) = \ln t + \beta\ln\left[1+\frac{\sigma^2}{t\beta} - \frac{1}{4}\mathcal{F}\left(\frac{\sigma^2}{t\beta},\beta\right)\right] + \ln\left[1+\frac{\sigma^2}{t} - \frac{1}{4}\mathcal{F}\left(\frac{\sigma^2}{t\beta},\beta\right)\right] - \frac{t\beta}{4\sigma^2}\mathcal{F}\left(\frac{\sigma^2}{t\beta},\beta\right)$$

- N.B.: This result is for cancellation only while the counterpart with suppression is already analyzed in our CISS'13-paper (ref. [12])

Mismatched Decoding: Per-stream Achievable Rate

- When $\boldsymbol{H}_{ij}$ and $\boldsymbol{H}_{jj}$ are i.i.d. Gaussian with gains $\bar{\gamma}_{ij}$ and $\bar{\gamma}_{jj}$ and the receiver postulates imperfectly $\tilde{\boldsymbol{R}}_{\boldsymbol{w}_j} = (1+\bar{\gamma}_{jj}\tilde{\sigma}_j^2)\boldsymbol{I}_N$:

$$\frac{R_{ij}}{M_i} = -\frac{s(1+\bar{\gamma}_{jj}\sigma_j^2)}{\alpha_{ij}(1+\bar{\gamma}_{jj}\tilde{\sigma}_j^2+s\tilde{E}_{ij})}\cdot\frac{s\tilde{E}_{ij}}{1+\bar{\gamma}_{jj}\tilde{\sigma}_j^2}+\ln\left(1+\frac{s\bar{\gamma}_{ij}}{\alpha_{ij}(1+\bar{\gamma}_{jj}\tilde{\sigma}_j^2+s\tilde{E}_{ij})}\right)+\frac{1}{\alpha_{ij}}\ln\left(1+\frac{s\tilde{E}_{ij}}{1+\bar{\gamma}_{jj}\tilde{\sigma}_j^2}\right)$$

where $\tilde{E}_{ij}$ is directly given as

$$\tilde{E}_{ij} = \frac{s\bar{\gamma}_{ij}-(1+\bar{\gamma}_{jj}\tilde{\sigma}_j^2)}{2s}-\frac{\bar{\gamma}_{ij}}{2\alpha_{ij}}+\sqrt{\frac{(1+\bar{\gamma}_{jj}\tilde{\sigma}_j^2)\bar{\gamma}_{ij}}{s}+\left(\frac{s\bar{\gamma}_{ij}-(1+\bar{\gamma}_{jj}\tilde{\sigma}_j^2)}{2s}-\frac{\bar{\gamma}_{ij}}{2\alpha_{ij}}\right)^2}$$

  - the case of $\tilde{\sigma}_j^2 = 0$ is illustrated in the numerical examples
  - asymptotic result at large-system limit: $M_i \to \infty$ and $N_j \to \infty$ while $\frac{M_i}{N_j} \to \alpha_{ij}$ for all $i, j \in \{1, 2\}$ (like in the previous slide)

- Optimization is required for the parameter s though, in order to find more tight lower bounds for the maximum achievable rate

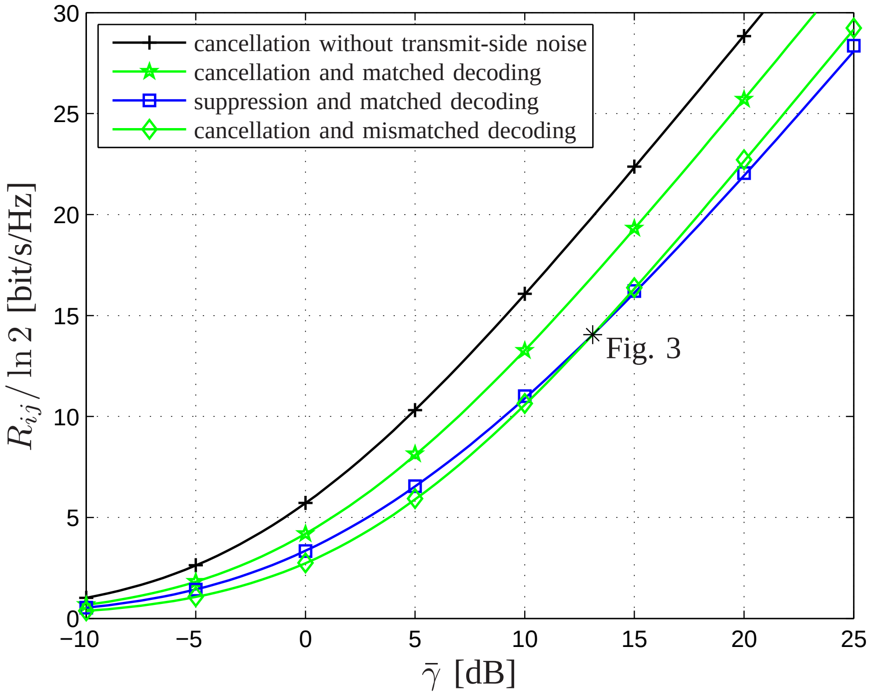
Numerical Examples

Example Setups

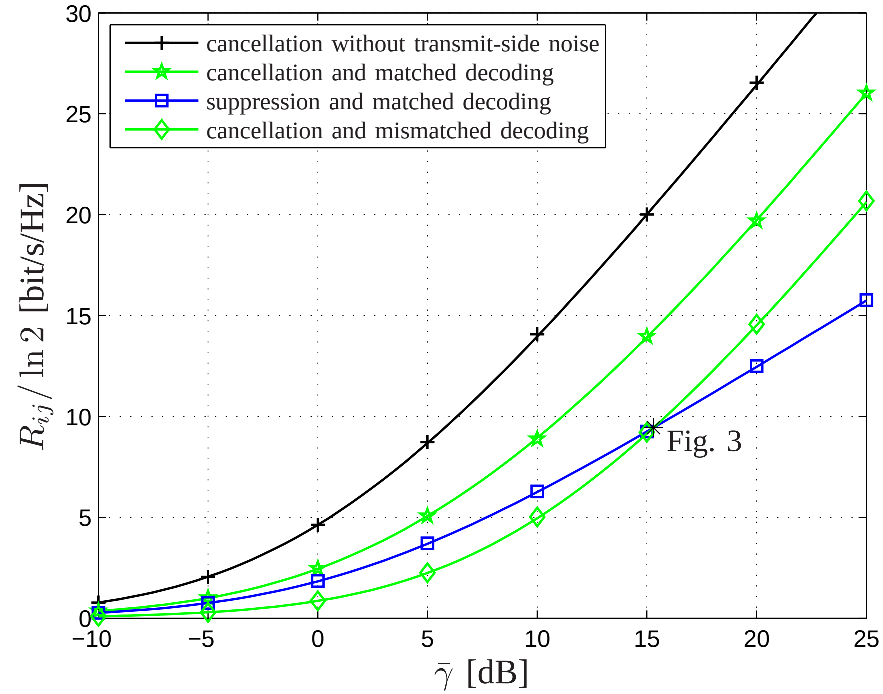
- The numerical results concentrate on symmetric systems where
  - $M = M_1 = M_2$
  - $N = N_1 = N_2$
  - $\bar{\gamma} = \bar{\gamma}_{12} = \bar{\gamma}_{21}$
  - $\bar{\gamma}_{\mathrm{I}} = \bar{\gamma}_{11} = \bar{\gamma}_{22}$
  - $\sigma^2 = \sigma_1^2 = \sigma_2^2$
- There may be transmit/receive antenna imbalance (M/N)
  - Yet M and N grow asymptotically at the large-system limit
- Choice $\sigma^2 = 0.001$ corresponds to transmitter EVM of -30 dB (or equivalently 3.2%) which is a practical but slightly optimistic value
- In summary, there are three key parameters to explore:

$$\boxed{\bar{\gamma}} \quad \boxed{\bar{\gamma}_{\mathrm{I}}} \quad \boxed{M/N}$$

Achievable Rates vs. SNR (Fig. 2)



(a) $M = 4$, $N = 8$, $\bar{\gamma}_{\mathrm{I}} = 33$ dB



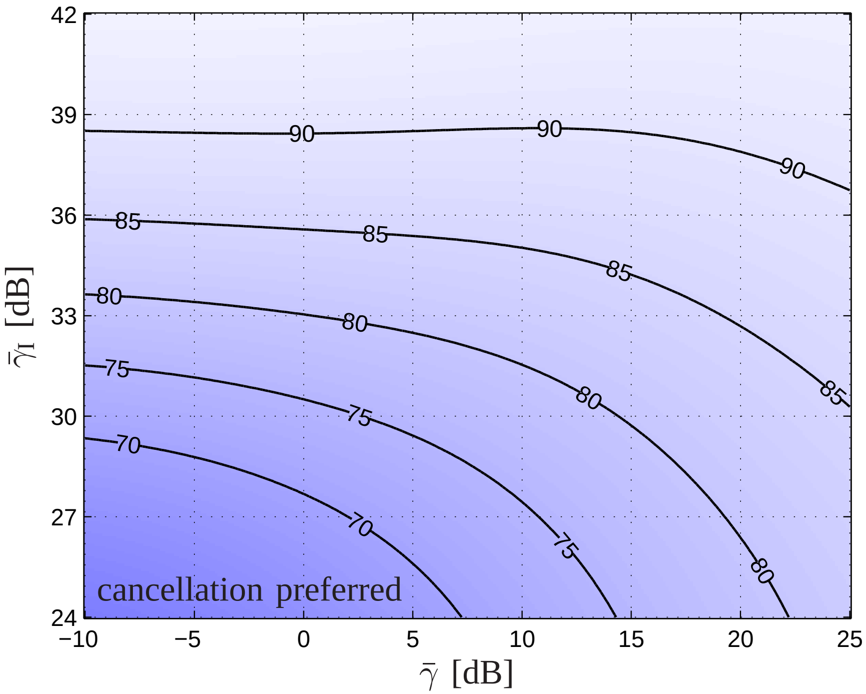
(b) $M = 4$, $N = 6$, $\bar{\gamma}_{\mathrm{I}} = 39$ dB

- Simulations (markers) corroborate analytical results (solid lines)

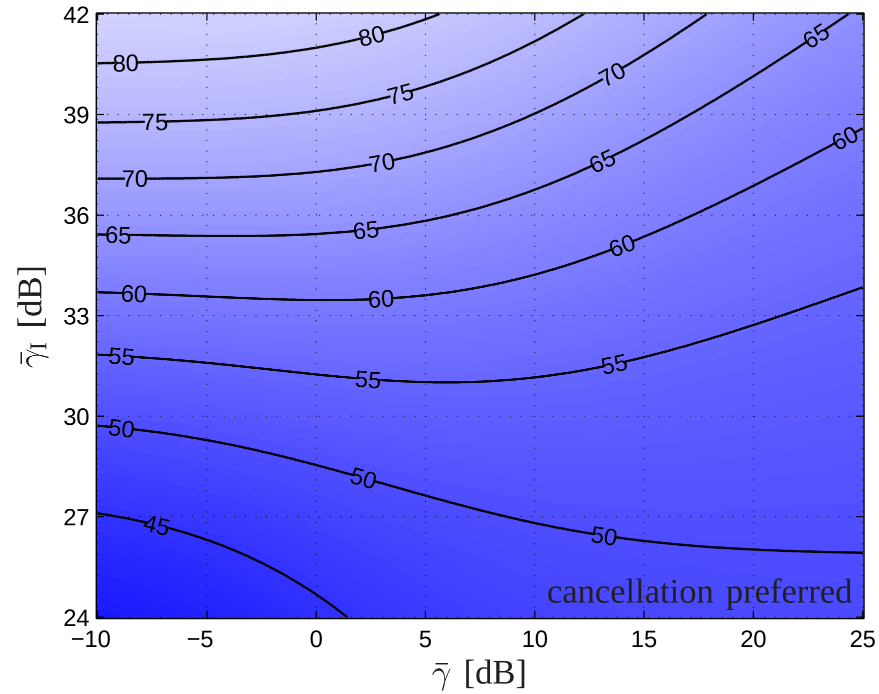
(a) when $M/N \leq 1/2$, suppression reduces receive array gain

(b) when $M/N > 1/2$, suppression reduces multiplexing order

Matched Decoding: Suppression vs. Cancellation [%]



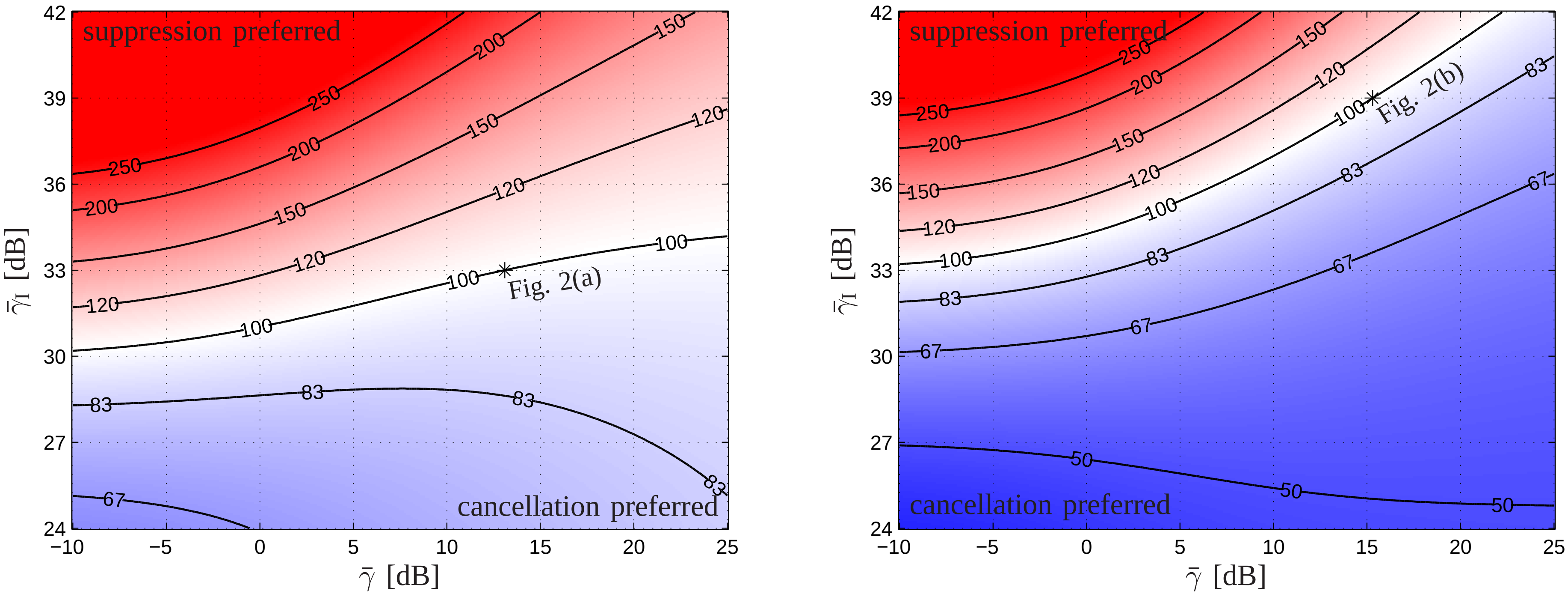
(a) $M/N = 1/2$ as in Fig. 2(a)



(b) $M/N = 2/3$ as in Fig. 2(b)

- Suppression is worse than cancellation if matched decoding is still feasible under residual self-interference, since such receivers already comprise ideal interference and noise control

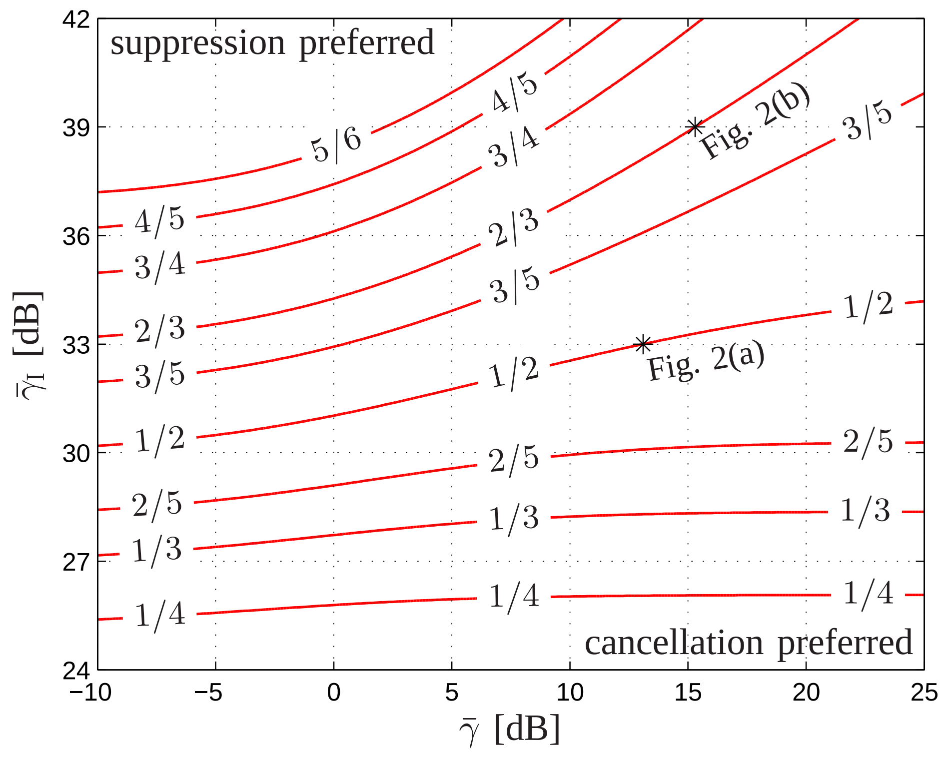
Mismatched Decoding: Suppression vs. Cancellation [%]



(a) $M/N = 1/2$ as in Fig. 2(a) (b) $M/N = 2/3$ as in Fig. 2(b)

- Transmitter noise and mismatched decoding cause an intricate interplay between the parameters corresponding to the channel gains of the data and self-interference links and the antenna ratio

Mismatched Decoding: Switching Boundaries (Fig. 3)



- Suppression becomes preferred in wide SNR range when the number of receive antennas vs. transmit antennas is large
- The level of self-interference is a significant factor at low SNR

Conclusion

Conclusion

- *Wireless full-duplex communication* becomes a hot research topic
  - ▷ A progressive frequency-reuse concept: significantly improved spectral efficiency at the cost of *self-interference*
- Achievable rates in bidirectional full-duplex link
  - ▷ *Mismatched decoding* due to *transmitter imperfections*
  - ▷ Analysis at the large-system limit based on the replica method
    - – Monte Carlo simulations with small number of antennas match well with the corresponding asymptotic results
- Comparison of spatial *suppression* and subtractive *cancellation*
  - ▷ for characterizing the cost and benefit of allocating a part of spatial degrees of freedom for self-interference mitigation
  - ▷ The study reveals a trade-off between reduced multiplexing order or array gain and residual self-interference

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