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Effect of Oscillator Phase Noise and Processing Delay in Full-Duplex OFDM Repeaters

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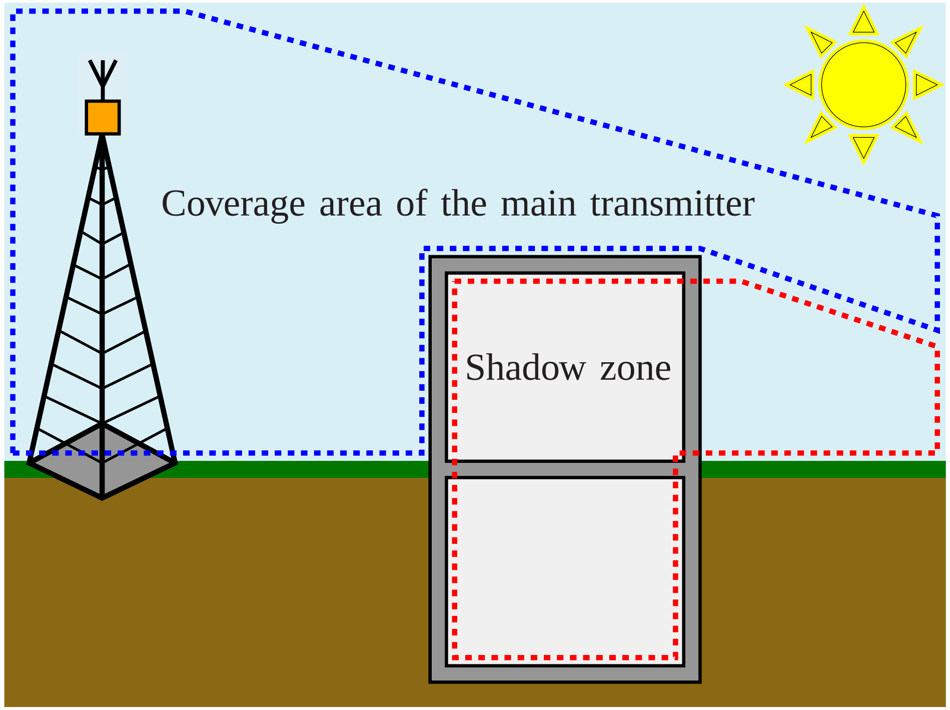
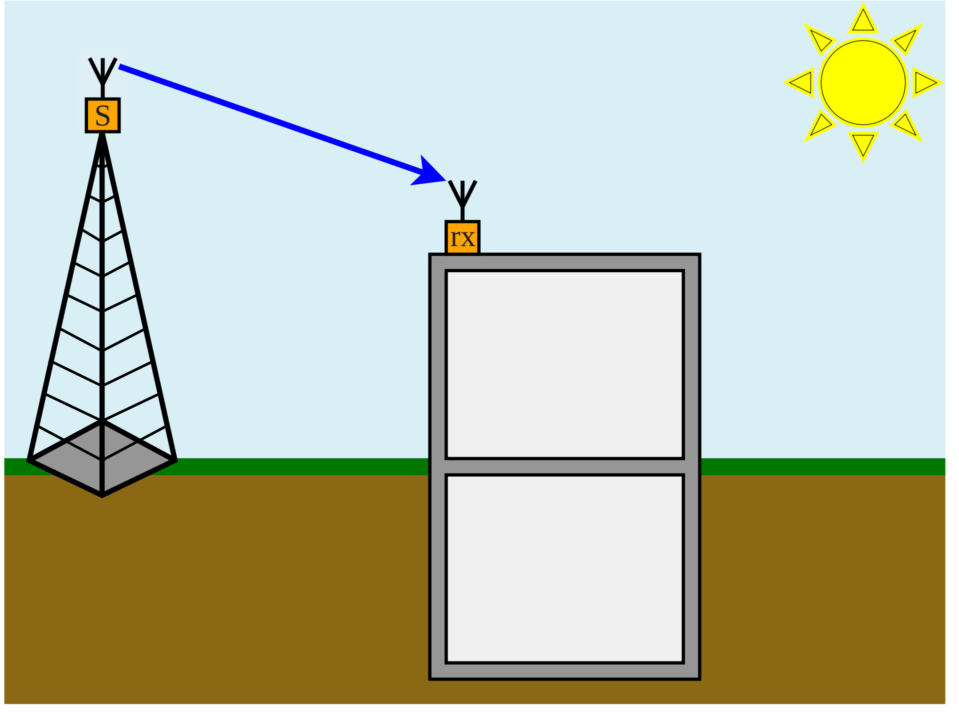
Introduction

Problem: Coverage Gaps



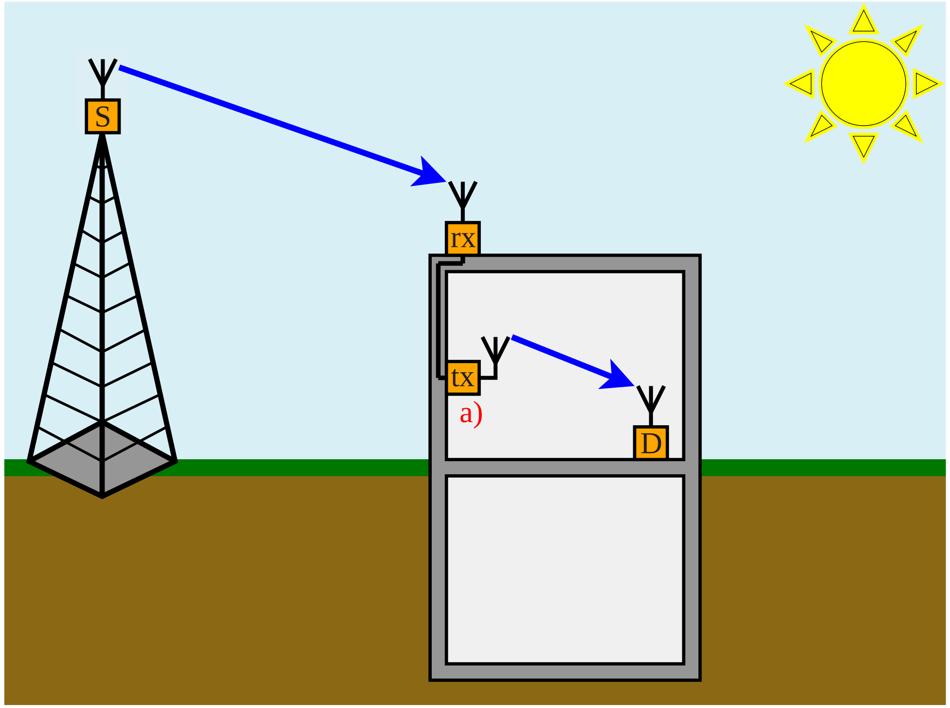
- How to serve shadowed areas in cellular systems?
  - ▷ Transmit powers cannot be increased indefinitely
  - ▷ The transmitter density needs to be higher and non-uniform

Solution: Full-Duplex Repeaters (1)



- Capture a good quality input signal within the main coverage area
  - ▷ highly directional receive (rx) antenna in an elevated position
  - ▷ preferably line-of-sight to the source (S) transmitter

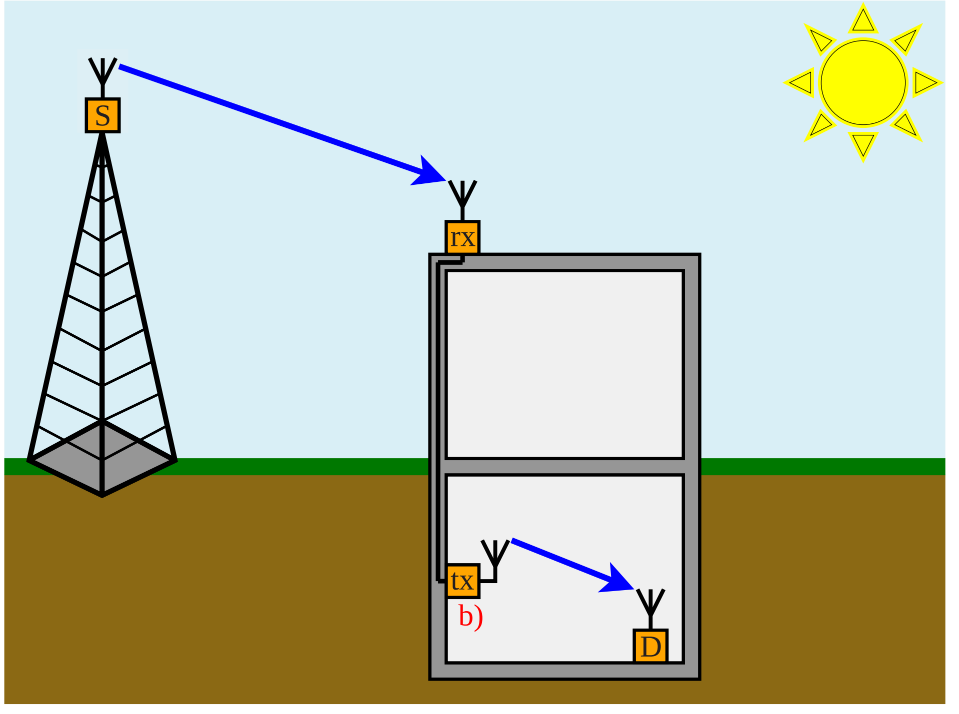
Solution: Full-Duplex Repeaters (2)



- Amplify and forward the signal within the shadow zone
- Omnidirectional transmit (tx) antenna, e.g., for providing

  a) indoor coverage

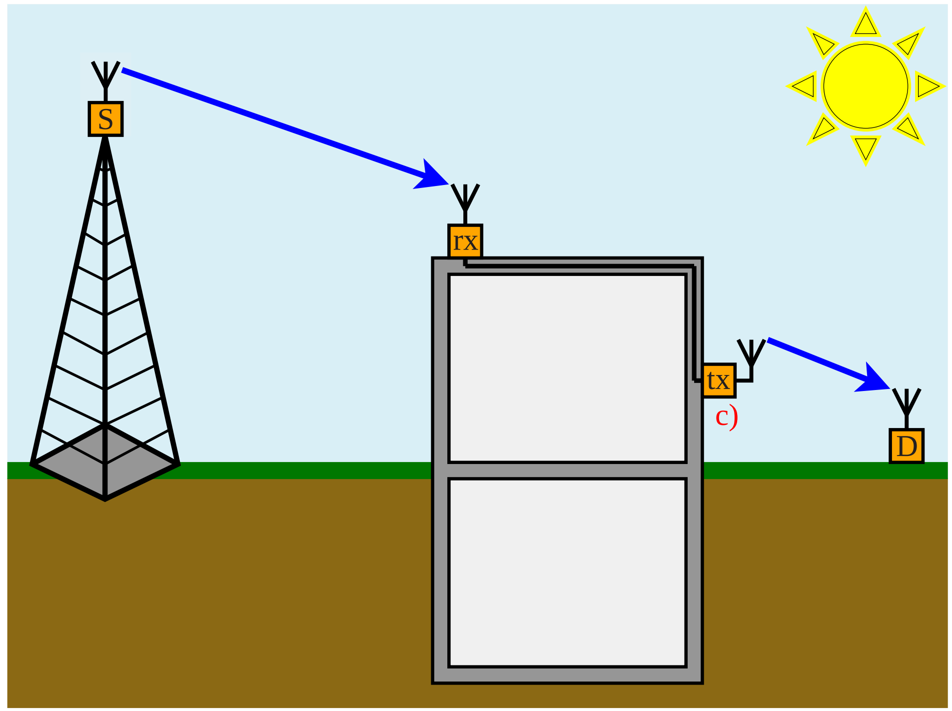
Solution: Full-Duplex Repeaters (3)



- Amplify and forward the signal within the shadow zone
- Omnidirectional transmit (tx) antenna, e.g., for providing

  b) underground coverage

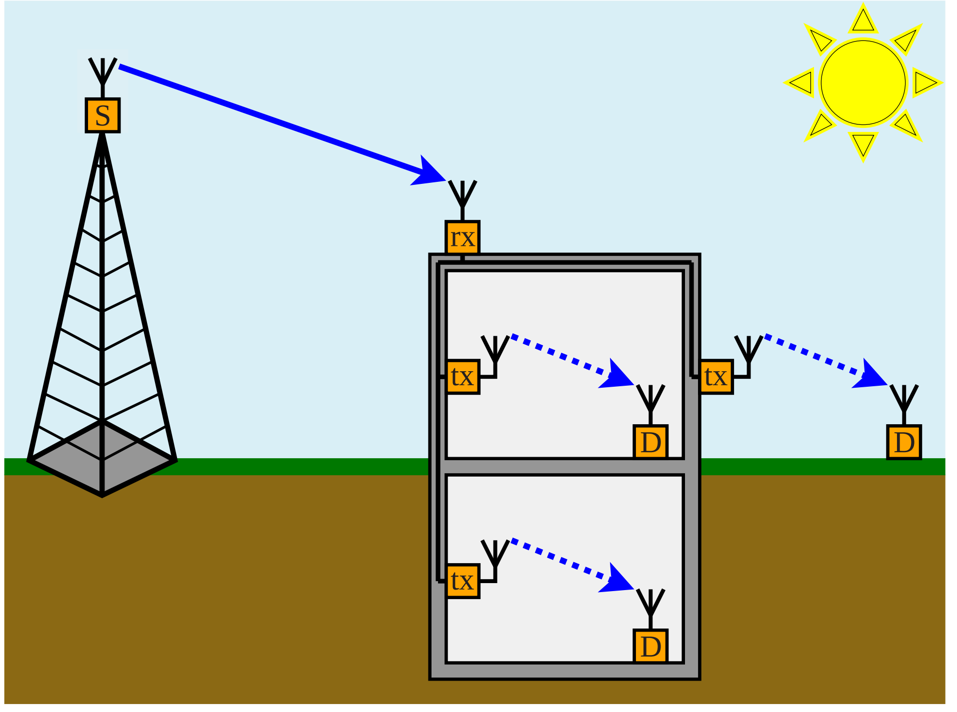
Solution: Full-Duplex Repeaters (4)



- Amplify and forward the signal within the shadow zone
- Omnidirectional transmit (tx) antenna, e.g., for providing

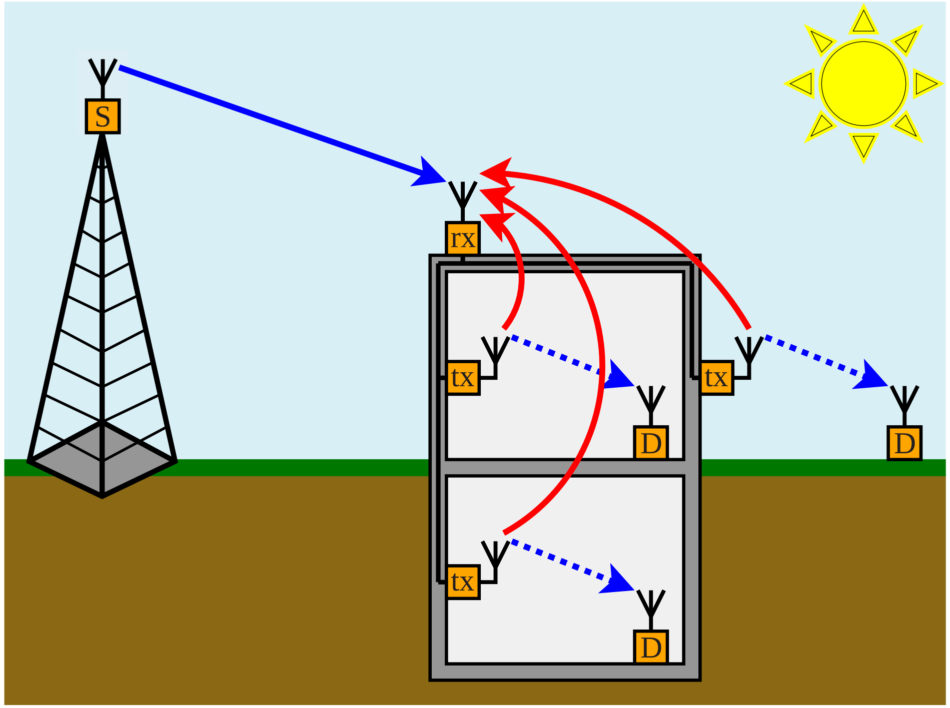
  c) coverage between buildings

Solution: Full-Duplex Repeaters (5)



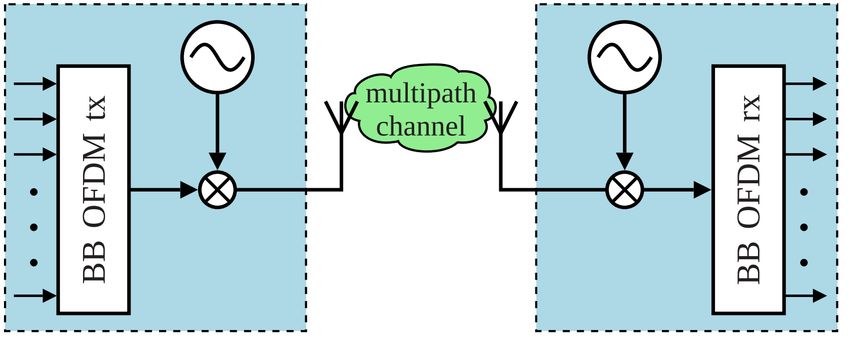
- Distributed tx antenna system can be also implemented
- Transparent coverage boost without allocating extra frequencies
- No wired (optical fiber) data connection needed, only power supply

Problem&Solution: Self-interference Cancellation



- Single-frequency operation comes at the cost of self-interference
- The repeater's gain needs to be limited to avoid oscillation
  - ▷ Herein: sufficient cancellation performance and gain margin

Problem: Oscillator Phase Noise in OFDM



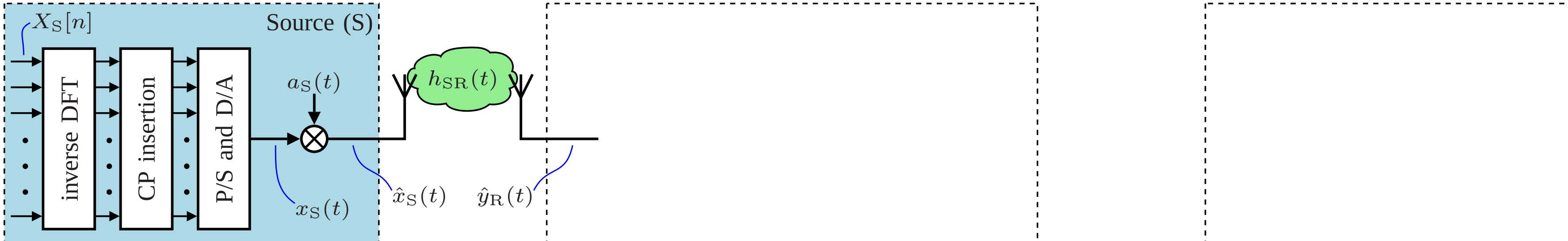
- Generally speaking, orthogonal frequency-division multiplexing is
  - robust to timing asynchronism and multipath delay spread
  - sensitive to *phase noise*, carrier offset, I/Q imbalance
- Jumps from base band (BB) to carrier frequency f_c and back to BB

$$\text{upconversion: } a_{tx}(t) = e^{\jmath 2\pi f_c t + \jmath\theta_{tx}(t)}$$
$$\text{downconversion: } a_{rx}(t) = e^{-\jmath 2\pi f_c t + \jmath\theta_{rx}(t)}$$

- Focus in this work: The effect of phase noise, $\theta_{tx}(t)$ and $\theta_{rx}(t)$, in terms of *processing delay* with two *different repeater designs*

System Model

OFDM Repeater Link: Signal Model (1)



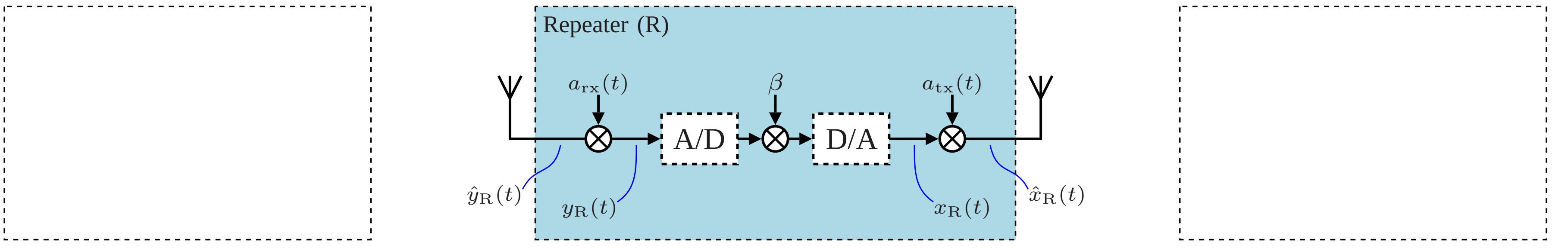
- Standard OFDM modulator: Frequency-domain symbols $\{X_{\mathrm{S}}[n]\}_{n=0}^{N_{\mathrm{c}}-1}$ are transformed into analog baseband signal $x_{\mathrm{S}}(t)$
- Upconversion: Mixing $x_{\mathrm{S}}(t)$ with oscillator signal $a_{\mathrm{S}}(t)$

$$\hat{x}_{\mathrm{S}}(t) = a_{\mathrm{S}}(t) \cdot x_{\mathrm{S}}(t)$$

  where the oscillator is assumed to be ideal: $a_{\mathrm{S}}(t) = e^{\jmath 2\pi f_{\mathrm{c}} t}$
- After a passband filter and a high-power amplifier, RF signal $\hat{x}_{\mathrm{S}}(t)$ propagates to the repeater through multipath channel $h_{\mathrm{SR}}(t)$

$$\hat{y}_{\mathrm{R}}(t) = (h_{\mathrm{SR}} * \hat{x}_{\mathrm{S}})(t) + \hat{w}_{\mathrm{R}}(t)$$

OFDM Repeater Link: Signal Model (2)



- Downconversion: Mixing $\hat{y}_R(t)$ with oscillator signal $a_{rx}(t)$

$$y_R(t) = a_{rx}(t) \cdot \hat{y}_R(t)$$

- Processing delay τ due to digital (or only analog?) filtering etc.
  - Amplification by β, self-interference cancellation, equalization
- Upconversion: Mixing $x_R(t)$ with oscillator signal $a_{tx}(t)$

$$\hat{x}_R(t) = a_{tx}(t) \cdot x_R(t)$$

- Non-ideal repeater oscillator(s): Phase noise in $a_{rx}(t)$ and $a_{tx}(t)$

OFDM Repeater Link: Signal Model (3)

- After a passband filter and a high-power amplifier, RF signal $\hat{x}_R(t)$ propagates to the destination through multipath channel $h_{RD}(t)$

$$\hat{y}_D(t) = (h_{RD} * \hat{x}_R)(t) + \hat{w}_D(t)$$

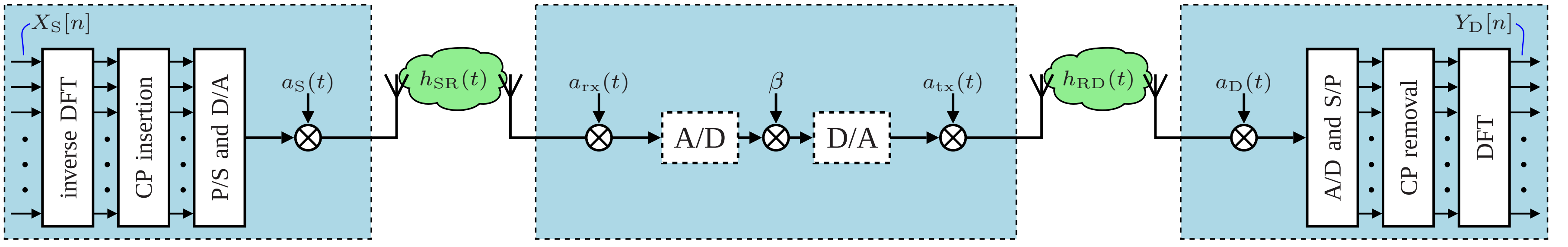
- Downconversion: Mixing $\hat{y}_D(t)$ with oscillator signal $a_D(t)$

$$y_D(t) = a_D(t) \cdot \hat{y}_D(t)$$

where the oscillator is assumed to be ideal: $a_D(t) = a_S^*(t)$

- Standard OFDM demodulator: Analog baseband signal $y_D(t)$ is transformed to frequency-domain symbols $\{Y_D[n]\}_{n=0}^{N_c-1}$

OFDM Repeater Link: Signal Model (4)



- Let us denote $a_R(t) = a_{tx}(t) \cdot a_{rx}(t-\tau)$
- End-to-end baseband signal model in time domain (simplified form):
  $y_D(t) = \beta h_{RD}(t) * \{a_R(t) \cdot (h_{SR} * x_S)(t-\tau) + w_R(t-\tau)\} + w_D(t)$
- Equivalent model in frequency domain for the nth subcarrier:

$$Y_D[n] = \beta H_{RD}[n] \sum_{k=0}^{N_c-1} A_R[k-n](H_{SR}[k]X_S[k]+W_R[k])+W_D[n]$$

  - Inter-carrier interference (ICI) is realized through $A_R[k]$ which corresponds to phasor $a_R(t)$ from oscillator phase noise

Signal-to-Interference and Noise Ratio (SINR)

- Signal, interference and noise powers

$$\mathcal{E}\{|Y_\mathrm{D}[n]|^2\} = \beta^2 |H_\mathrm{RD}[n]|^2 \sum_{k=0}^{N_\mathrm{c}-1} |A_\mathrm{R}[k-n]|^2 (|H_\mathrm{SR}[k]|^2 P_\mathrm{S}[k] + \sigma_\mathrm{R}^2) + \sigma_\mathrm{D}^2$$

  where $P_\mathrm{S}[n] = \mathcal{E}\{|X_\mathrm{S}[n]|^2\}$, $\sigma_\mathrm{R}^2 = \mathcal{E}\{|W_\mathrm{R}[n]|^2\}$, $\sigma_\mathrm{D}^2 = \mathcal{E}\{|W_\mathrm{D}[n]|^2\}$

- With sufficiently coherent channels (vs. oscillator's spectral density)

$$\mathcal{E}\{|Y_\mathrm{D}[n]|^2\} \simeq \beta^2 |H_\mathrm{RD}[n]|^2 (|H_\mathrm{SR}[n]|^2 P_\mathrm{S}[n] + \sigma_\mathrm{R}^2) \sum_{k=0}^{N_\mathrm{c}-1} |A_\mathrm{R}[k]|^2 + \sigma_\mathrm{D}^2$$

- Finally, the instantaneous SINR can be expressed as

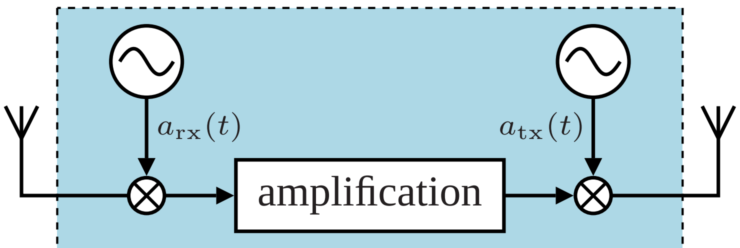
$$\gamma[n] = \frac{(1-\alpha)\gamma_\mathrm{SR}[n]\gamma_\mathrm{RD}[n]}{(\alpha\,\gamma_\mathrm{SR}[n]+1)\gamma_\mathrm{RD}[n] + \frac{P_\mathrm{tx}[n]}{\sigma_\mathrm{R}^2\beta^2}}$$

  where $\alpha = 1 - |A_\mathrm{R}[0]|^2 = \sum_{k=1}^{N_\mathrm{c}-1} |A_\mathrm{R}[k]|^2$ represents *ICI power* and SNRs are $\gamma_\mathrm{SR}[n] = P_\mathrm{S}[n]|H_\mathrm{SR}[n]|^2/\sigma_\mathrm{R}^2$, $\gamma_\mathrm{RD}[n] = P_\mathrm{tx}[n]|H_\mathrm{RD}[n]|^2/\sigma_\mathrm{D}^2$

Non-ideal Oscillators in the Repeater

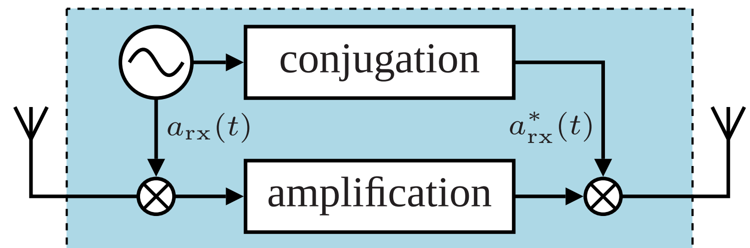
- In the following: Comparison of two different repeater designs

(a) Two separate oscillators



- downconversion:
  $a_{\mathrm{rx}}(t) = e^{-\jmath(2\pi f_{\mathrm{c}} t - \theta_{\mathrm{rx}}(t))}$
- upconversion:
  $a_{\mathrm{tx}}(t) = e^{\jmath(2\pi f_{\mathrm{c}} t + \theta_{\mathrm{tx}}(t))}$

(b) Reusing single oscillator



- downconversion:
  $a_{\mathrm{rx}}(t) = e^{-\jmath(2\pi f_{\mathrm{c}} t - \theta_{\mathrm{rx}}(t))}$
- upconversion:
  $a_{\mathrm{tx}}(t) = a^*_{\mathrm{rx}}(t) = e^{\jmath(2\pi f_{\mathrm{c}} t - \theta_{\mathrm{rx}}(t))}$

- The total phase distortion caused by phase noise and repeater *processing delay* τ can be captured as phasor process

$$a_{\mathrm{R}}(t) = a_{\mathrm{rx}}(t-\tau) \cdot a_{\mathrm{tx}}(t)$$

Wiener Phase Noise

- The phase noise of free-running oscillators can be modelled accurately as a Wiener process, i.e., standard Brownian motion or “random walk with Gaussian steps”:

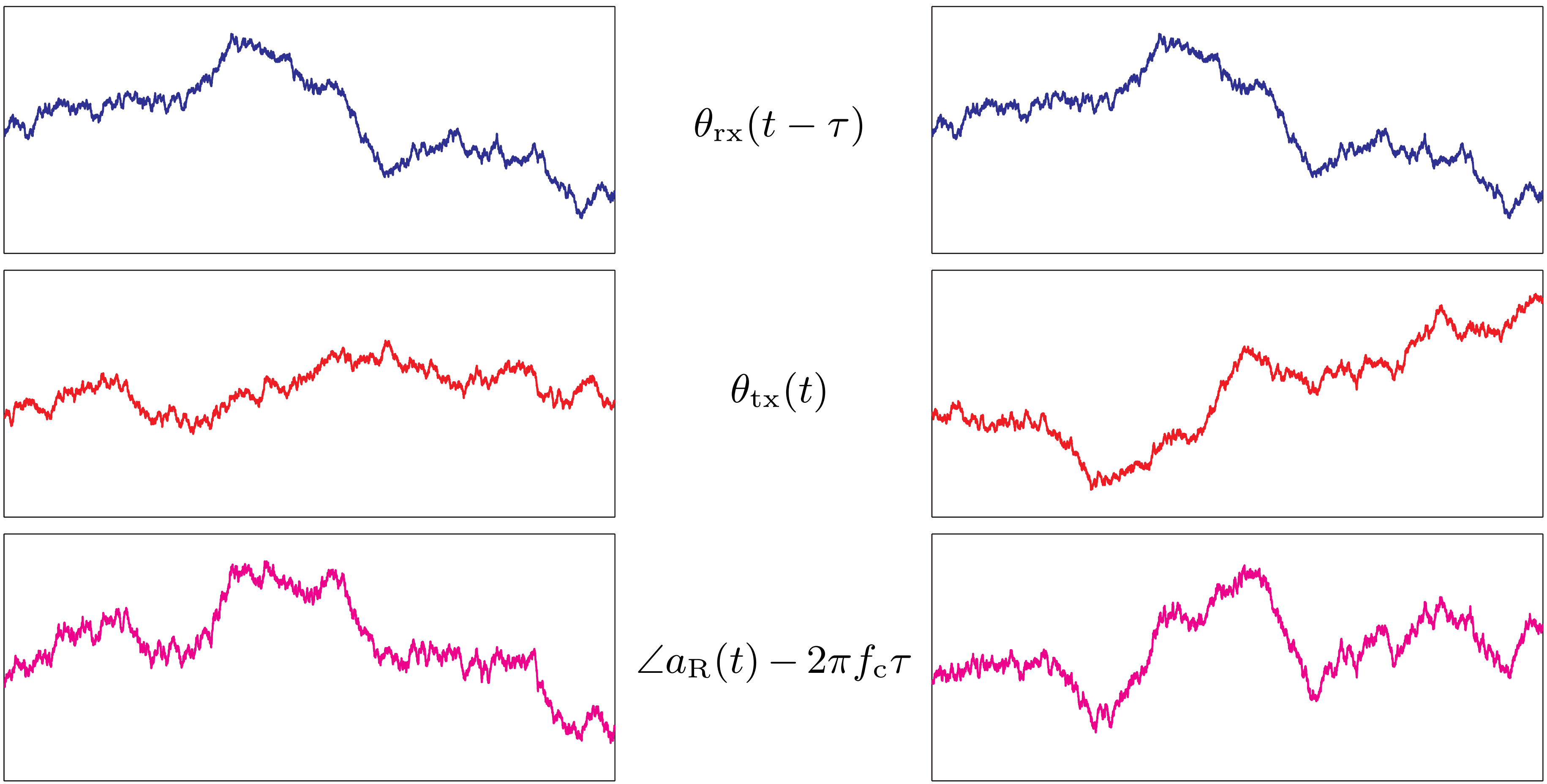
$$\theta_{\mathrm{rx}}(t_0) - \theta_{\mathrm{rx}}(t_0 - t) \sim \mathcal{N}(0, c_{\mathrm{rx}} \cdot |t|)$$
$$\theta_{\mathrm{tx}}(t_0) - \theta_{\mathrm{tx}}(t_0 - t) \sim \mathcal{N}(0, c_{\mathrm{tx}} \cdot |t|)$$

- (In)dependence of rx and tx sides due to the repeater design
  - ▷ Two separate oscillators: $\theta_{\mathrm{tx}}(t)$ is independent of $\theta_{\mathrm{rx}}(t)$
  - ▷ Reusing single oscillator: $\theta_{\mathrm{tx}}(t) = -\theta_{\mathrm{rx}}(t)$
- The quality of the oscillator is parametrized by $f_{3\mathrm{dB}}$ which defines the 3dB bandwidth of the oscillator power spectral density (PSD)
  - ▷ When using two oscillators, they are assumed to be of similar quality in this study: $c = c_{\mathrm{rx}} = c_{\mathrm{tx}} = 4\pi f_{3\mathrm{dB}}$

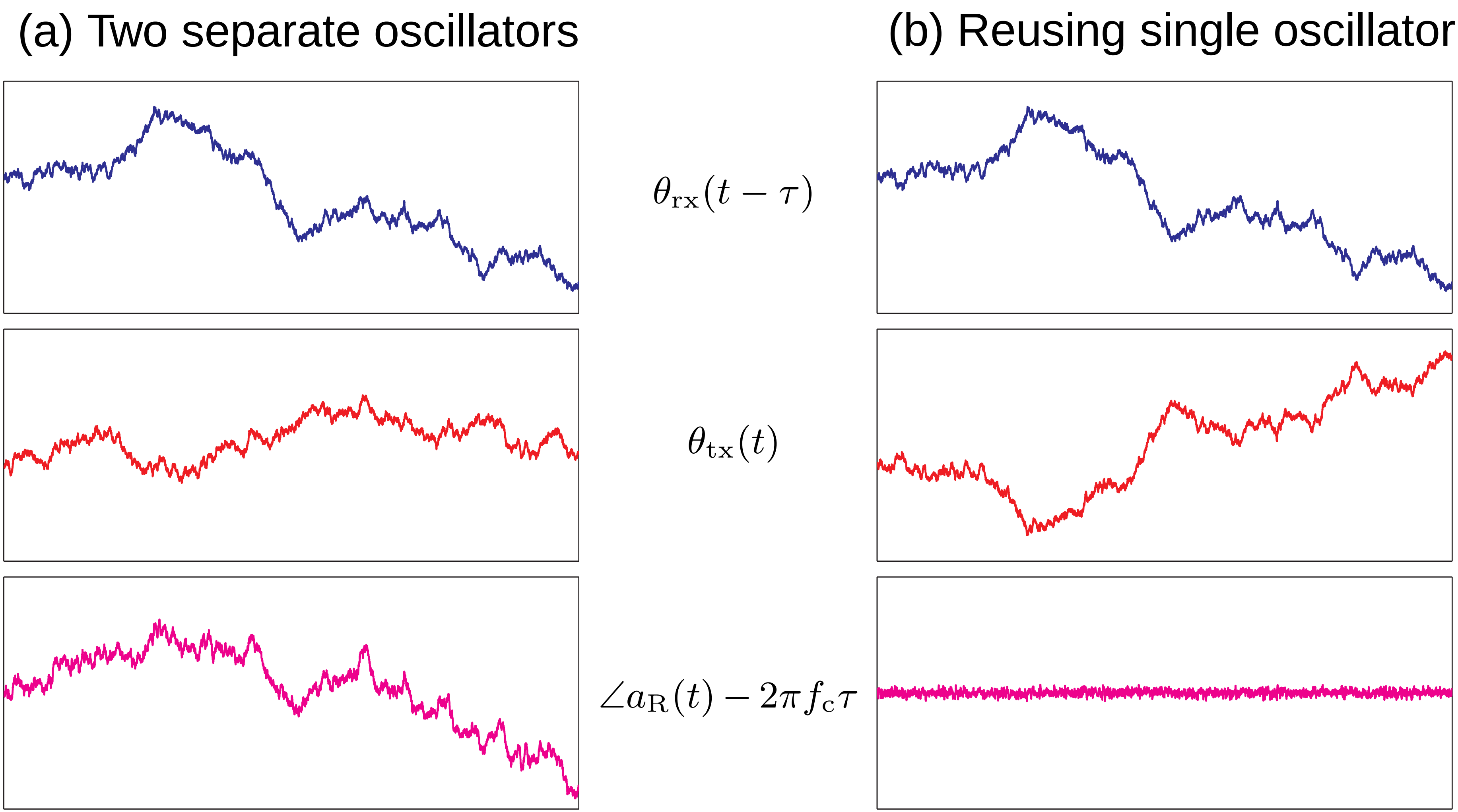
Spectral Spreading due to Phase Noise

Example (1): Long Processing Delay

(a) Two separate oscillators | (b) Reusing single oscillator



Example (2): Short Processing Delay



PSD of Repeater Phasor Process

- Total phase distortion caused by the repeater
$$a_{\mathrm{R}}(t) = a_{\mathrm{rx}}(t-\tau)\cdot a_{\mathrm{tx}}(t) = \begin{cases} e^{\jmath(2\pi f_{\mathrm{c}}\tau+\theta_{\mathrm{rx}}(t-\tau)+\theta_{\mathrm{tx}}(t))}, & \text{two oscillators} \\ e^{\jmath(2\pi f_{\mathrm{c}}\tau+\theta_{\mathrm{rx}}(t-\tau)-\theta_{\mathrm{rx}}(t))}, & \text{one oscillator} \end{cases}$$
- ICI is realized through $A_{\mathrm{R}}[k]$ which represents instantaneous spectral spreading for each OFDM symbol
- Standard steps for calculating power spectral density (PSD): first $R(t) = \mathcal{E}\{a_{\mathrm{R}}(t_0)a_{\mathrm{R}}^*(t_0-t)\}$ then $S(f) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R(t)e^{-\jmath 2\pi f t}\,\mathrm{d}t$
  - ▷ PSD is related to $\mathcal{E}\{|A_{\mathrm{R}}[k]|^2\}$, i.e., expected ICI power
  - ▷ In the ideal case $a_{\mathrm{R}}(t) = e^{\jmath 2\pi f_{\mathrm{c}}\tau}$ yielding $S_0(f) = \delta(f)$

(a) Two separate oscillators

$$S_2(f) = \frac{1}{\pi}\cdot\frac{c}{c^2+(2\pi f)^2}$$

(b) Reusing single oscillator

$$S_1(f) = e^{-c\tau}S_0(f) + \tilde{S}_1(f)\,S_2(f)$$

where

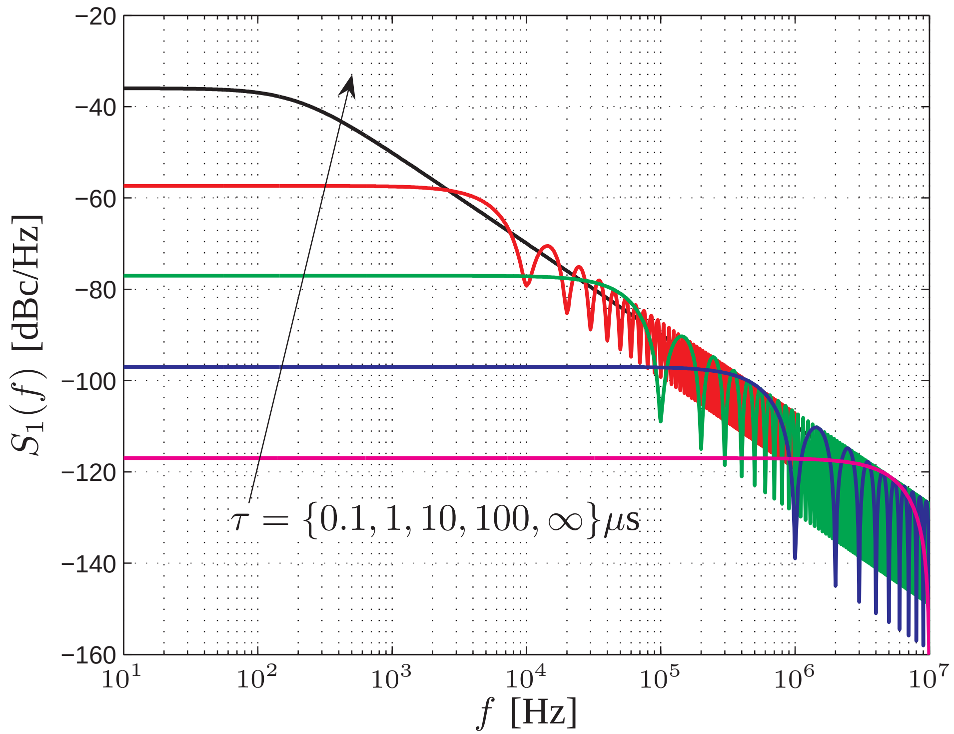
$$\tilde{S}_1(f) = 1 - e^{-c\tau}\left(\cos(2\pi f\tau) + c\tau\,\mathrm{sinc}(2\pi f\tau)\right)$$

Numerical Results (1)

- On the right: PSD when reusing single oscillator and $f_{3\text{dB}} = 100\text{Hz}$
- Extreme cases for processing delay:

  $$S_1(f) \to \begin{cases} S_0(f), & \text{when } \tau \to 0 \\ S_2(f), & \text{when } \tau \to \infty \end{cases}$$

  (cf. OFDM sample and symbol duration)



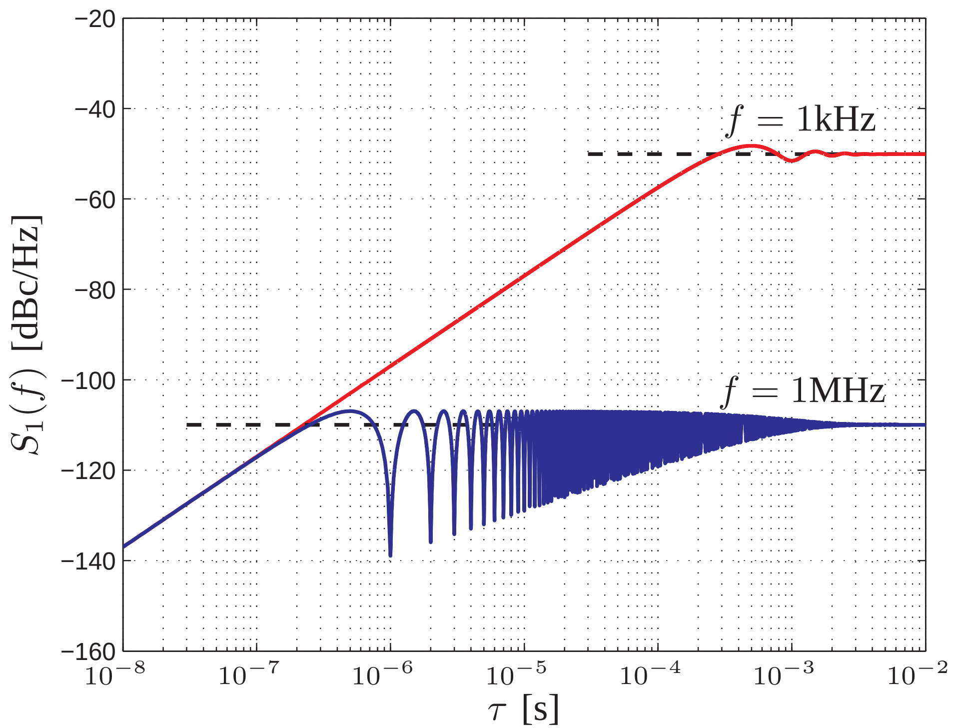
- Except for the impulse at the zero frequency (not visible in the above figure), the PSD is approximately flat when $f < \frac{1}{4\tau}$
- When $f > \frac{1}{4\tau}$, the PSD decays 20dB per decade

Numerical Results (2)

- On the right: PSD vs. processing delay when reusing single oscillator and $f_{3\mathrm{dB}} = 100\mathrm{Hz}$

  at 1kHz: Flat PSD
  at 1MHz: PSD decays
  20 dB per decade

  (cf. subcarrier spacing and bandwidth in OFDM)



- The PSD oscillates less when the processing delay increases and becomes smooth when $\tau > \frac{1}{4f_{3\mathrm{dB}}}$ (“$\approx \infty$”)
  - ▷ However, OFDM symbols are typically shorter than that

Transmission Rate Analysis

Distribution of ICI Power

- When reusing a single oscillator, time-domain phase distortion $\angle a_{\mathrm{R}}(t)$ can be seen as colored Gaussian noise
- And $A_{\mathrm{R}}[k]$ represents $a_{\mathrm{R}}(t)$ in frequency domain after sampling
- Using Taylor series expansion, ICI power $\alpha = \sum_{k=1}^{N_{\mathrm{c}}-1} |A_{\mathrm{R}}[k]|^2$ becomes a sum of correlated gamma random variables
  - ▷ Coefficients λ_k depend on $f_{3\mathrm{dB}}$ and τ via a covariance matrix!
- Finally, the probability density function (PDF) of α can be expressed as a weighted sum of gamma PDFs:

$$p(\alpha) = \kappa \sum_{k=0}^{\infty} \zeta_k \, p_k(\alpha) \quad \text{where} \quad p_k(\alpha) = \frac{\alpha^{\frac{N_{\mathrm{c}}-1}{2}+k-1} e^{-\frac{\alpha}{\lambda_1}}}{\lambda_1^{\frac{N_{\mathrm{c}}-1}{2}+k} \Gamma\left(\frac{N_{\mathrm{c}}-1}{2}+k\right)}$$

  - ▷ $\kappa = \prod_{n=1}^{N_{\mathrm{c}}-1} \sqrt{\frac{\lambda_1}{\lambda_n}}$ (probability mass normalization)
  - ▷ $\zeta_0 = 1$ and $\zeta_{k+1} = \frac{1/2}{k+1} \sum_{i=1}^{k+1} \sum_{j=1}^{N_{\mathrm{c}}-1} \left(1 - \lambda_1/\lambda_j\right)^i \zeta_{k+1-i}$

Average Transmission Rate

- Repeater gain $\beta^2 = (P_{\mathrm{tx}}[n]/\sigma_{\mathrm{R}}^2)/(\gamma_{\mathrm{SR}}[n]+1)$ transforms SINR to

$$\gamma[n] = \frac{(1-\alpha)\gamma_{\mathrm{SR}}[n]\gamma_{\mathrm{RD}}[n]}{\gamma_{\mathrm{SR}}[n] + (\alpha\,\gamma_{\mathrm{SR}}[n]+1)\gamma_{\mathrm{RD}}[n] + 1}$$

- Instantaneous transmission rate is given by

$$C[n] = \log_2(1+\gamma[n]) = \log_2\left(\frac{\gamma_{\mathrm{SR}}[n]\gamma_{\mathrm{RD}}[n] + \gamma_{\mathrm{SR}}[n] + \gamma_{\mathrm{RD}}[n] + 1}{\alpha\gamma_{\mathrm{SR}}[n]\gamma_{\mathrm{RD}}[n] + \gamma_{\mathrm{SR}}[n] + \gamma_{\mathrm{RD}}[n] + 1}\right)$$

- Using $p(\alpha)$, average transmission rate can be calculated as

$$\bar{C}[n] = \mathcal{E}\{C[n]\} = \log_2\left(1 + \frac{\gamma_{\mathrm{SR}}[n]\gamma_{\mathrm{RD}}[n]}{\gamma_{\mathrm{SR}}[n] + \gamma_{\mathrm{RD}}[n] + 1}\right) - \kappa\sum_{k=0}^{\infty}\zeta_k\,\mathcal{I}_k$$

where $\mathcal{I}_k = \int_0^\infty \log_2\left(1 + \frac{\gamma_{\mathrm{SR}}[n]\gamma_{\mathrm{RD}}[n]}{\gamma_{\mathrm{SR}}[n] + \gamma_{\mathrm{RD}}[n] + 1}\alpha\right) p_k(\alpha)\,\mathrm{d}\alpha$

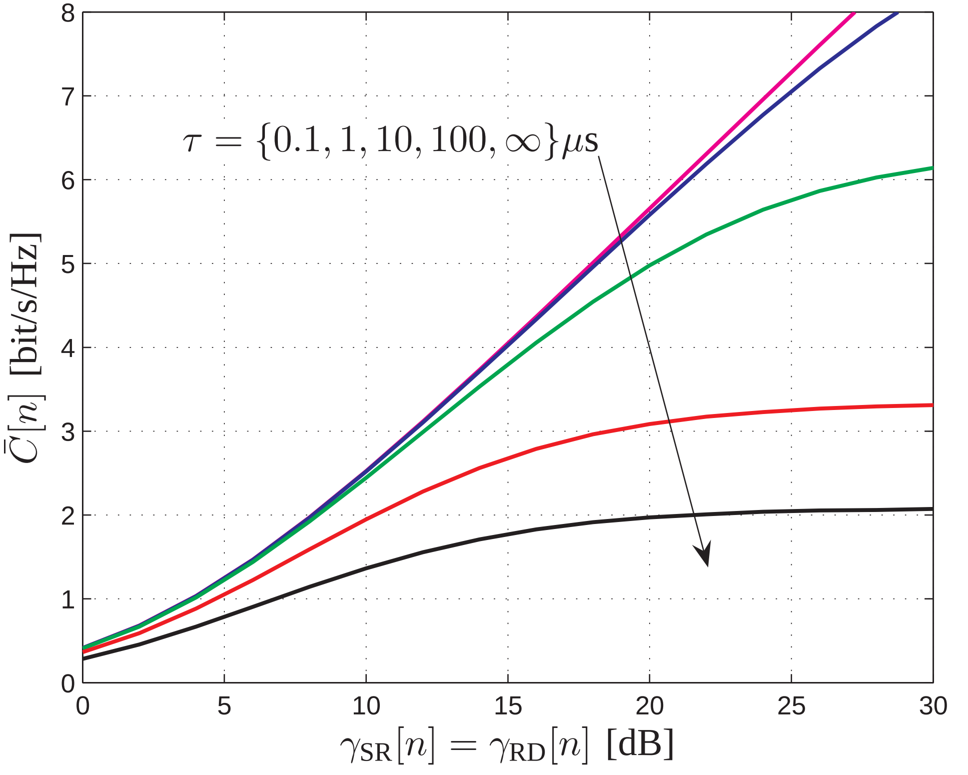
  - ▷ $\mathcal{I}_k$ can be solved in a closed form using Meijer's G-function (or generalized hypergeometric and incomplete gamma functions)

Numerical Results (3)

- OFDM parameters for a DVB-T/H-like system:

  $N_\mathrm{c} = 8192$ subcarriers
  and $8\mathrm{MHz}$ bandwidth
  – sample duration: $0.11\mu\mathrm{s}$
  – FFT duration: $896\mu\mathrm{s}$
  – subcarrier spacing: $1.1\mathrm{kHz}$

- Oscillators: $f_{3\mathrm{dB}} = 100\mathrm{Hz}$



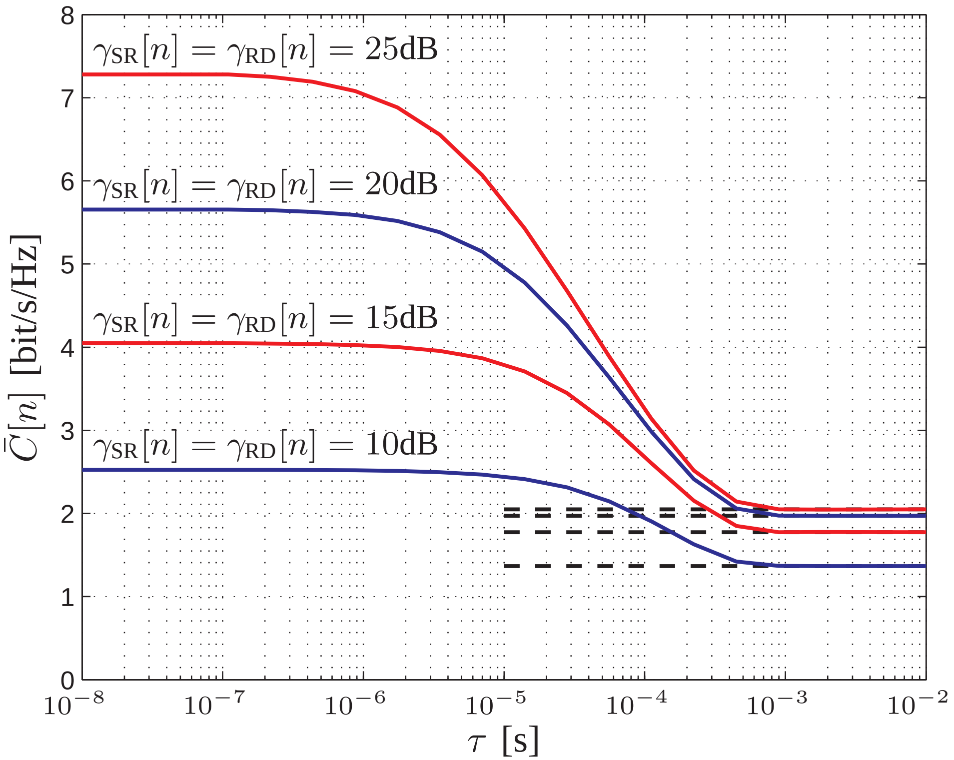
- When reusing a single oscillator, transmission rate degradation can be minimized by decreasing the processing delay
  - ▷ Implementation with two separate oscillators means $\tau \to \infty$

Numerical Results (4)

- OFDM parameters for a DVB-T/H-like system:

  $N_c = 8192$ subcarriers
  and 8MHz bandwidth
  – sample duration: $0.11\mu\text{s}$
  – FFT duration: $896\mu\text{s}$
  – subcarrier spacing: 1.1kHz

- Oscillators: $f_{3\text{dB}} = 100\text{Hz}$



- If the processing delay is a few tens of OFDM samples or shorter, the transmit-side noise reverts the effect of receive-side noise
  - ▷ The delay needs to be shorter than the cyclic prefix anyway

Conclusion

Conclusion

- Target: To understand the effect of spectral spreading on a full-duplex OFDM repeater link due to imperfect oscillator(s)
  - ▷ Phase noise causes inter-carrier interference (ICI)
- Comparison of two different repeater designs
  1. Using a single oscillator signal for down- and upconversion: *Processing delay becomes a key factor for spectral spreading!*
  2. Separate oscillators for down- and upconversion
- Analysis and numerical results at three abstraction levels
  1. Time-domain phase noise realizations vs. processing delay
  2. Power spectral density of repeater's phase distortion process
  3. Distribution of ICI power, and average transmission rate
- The transmit-side phase noise *can* partially revert the effect of receive-side distortion when processing delay is short enough

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