Effect of Oscillator Phase Noise and Processing Delay in Full-Duplex OFDM Repeaters

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Introduction
Problem: Coverage Gaps

- How to serve shadowed areas in cellular systems?
  - Transmit powers cannot be increased indefinitely
  - The transmitter density needs to be higher and non-uniform
Solution: Full-Duplex Repeaters (1)

- Capture a good quality input signal within the main coverage area
  - highly directional receive (rx) antenna in an elevated position
  - preferably line-of-sight to the source (S) transmitter
Solution: Full-Duplex Repeaters (2)

- Amplify and forward the signal within the shadow zone
- Omnidirectional transmit (tx) antenna, e.g., for providing
  
  a) indoor coverage
Solution: Full-Duplex Repeaters (3)

- Amplify and forward the signal within the shadow zone
- Omnidirectional transmit (tx) antenna, e.g., for providing
  b) underground coverage
Solution: Full-Duplex Repeaters (4)

- Amplify and forward the signal within the shadow zone
- Omnidirectional transmit (tx) antenna, e.g., for providing
  c) coverage between buildings
Solution: Full-Duplex Repeaters (5)

- Distributed tx antenna system can be also implemented
- Transparent coverage boost without allocating extra frequencies
- No wired (optical fiber) data connection needed, only power supply
Problem & Solution: Self-interference Cancellation

- Single-frequency operation comes at the cost of self-interference
- The repeater’s gain needs to be limited to avoid oscillation
  - Herein: sufficient cancellation performance and gain margin
Problem: Oscillator Phase Noise in OFDM

- Generally speaking, orthogonal frequency-division multiplexing is
  - robust to timing asynchronism and multipath delay spread
  - sensitive to phase noise, carrier offset, I/Q imbalance
- Jumps from base band (BB) to carrier frequency $f_c$ and back to BB
  - upconversion: $a_{tx}(t) = e^{j2\pi f_c t + j\theta_{tx}(t)}$
  - downconversion: $a_{rx}(t) = e^{-j2\pi f_c t + j\theta_{rx}(t)}$
- **Focus in this work:** The effect of phase noise, $\theta_{tx}(t)$ and $\theta_{rx}(t)$, in terms of processing delay with two different repeater designs
System Model
**OFDM Repeater Link: Signal Model (1)**

- **Standard OFDM modulator**: Frequency-domain symbols \( \{X_S[n]\}_{n=0}^{N_c-1} \) are transformed into analog baseband signal \( x_S(t) \).
- **Upconversion**: Mixing \( x_S(t) \) with oscillator signal \( a_S(t) \)

\[
\hat{x}_S(t) = a_S(t) \cdot x_S(t)
\]

where the oscillator is assumed to be ideal: \( a_S(t) = e^{j2\pi f_c t} \).
- **After a passband filter and a high-power amplifier**, RF signal \( \hat{x}_S(t) \) propagates to the repeater through multipath channel \( h_{SR}(t) \)

\[
\hat{y}_R(t) = (h_{SR} \ast \hat{x}_S)(t) + \hat{w}_R(t)
\]
- Downconversion: Mixing $\hat{y}_R(t)$ with oscillator signal $a_{rx}(t)$

$$y_R(t) = a_{rx}(t) \cdot \hat{y}_R(t)$$

- Processing delay $\tau$ due to digital (or only analog?) filtering etc.

  ▶ Amplification by $\beta$, self-interference cancellation, equalization

- Upconversion: Mixing $x_R(t)$ with oscillator signal $a_{tx}(t)$

$$\hat{x}_R(t) = a_{tx}(t) \cdot x_R(t)$$

- Non-ideal repeater oscillator(s): Phase noise in $a_{rx}(t)$ and $a_{tx}(t)$
After a passband filter and a high-power amplifier, RF signal $\hat{x}_R(t)$ propagates to the destination through multipath channel $h_{RD}(t)$

$$\hat{y}_D(t) = (h_{RD} * \hat{x}_R)(t) + \hat{w}_D(t)$$

Downconversion: Mixing $\hat{y}_D(t)$ with oscillator signal $a_D(t)$

$$y_D(t) = a_D(t) \cdot \hat{y}_D(t)$$

where the oscillator is assumed to be ideal: $a_D(t) = a_S^*(t)$

Standard OFDM demodulator: Analog baseband signal $y_D(t)$ is transformed to frequency-domain symbols $\{Y_D[n]\}_{n=0}^{N_c-1}$
OFDM Repeater Link: Signal Model (4)

- Let us denote $a_R(t) = a_{tx}(t) \cdot a_{rx}(t - \tau)$
- End-to-end baseband signal model in time domain (simplified form):
  $$y_D(t) = \beta h_{RD}(t) \ast \{a_R(t) \cdot (h_{SR} \ast x_S)(t - \tau) + w_R(t - \tau)\} + w_D(t)$$
- Equivalent model in frequency domain for the $n$th subcarrier:
  $$Y_D[n] = \beta H_{RD}[n] \sum_{k=0}^{N_c-1} A_R[k-n](H_{SR}[k]X_S[k] + W_R[k]) + W_D[n]$$
  ▶ Inter-carrier interference (ICI) is realized through $A_R[k]$ which corresponds to phasor $a_R(t)$ from oscillator phase noise
Signal-to-Interference and Noise Ratio (SINR)

- Signal, interference and noise powers

\[ E\{|Y_D[n]|^2\} = \beta^2 |H_{RD}[n]|^2 \sum_{k=0}^{N_c-1} |A_R[k-n]|^2 (|H_{SR}[k]|^2 P_S[k] + \sigma^2_R) + \sigma^2_D \]

where \( P_S[n] = E\{|X_S[n]|^2\} \), \( \sigma^2_R = E\{|W_R[n]|^2\} \), \( \sigma^2_D = E\{|W_D[n]|^2\} \)

- With sufficiently coherent channels (vs. oscillator’s spectral density)

\[ E\{|Y_D[n]|^2\} \simeq \beta^2 |H_{RD}[n]|^2 (|H_{SR}[n]|^2 P_S[n] + \sigma^2_R) \sum_{k=0}^{N_c-1} |A_R[k]|^2 + \sigma^2_D \]

- Finally, the instantaneous SINR can be expressed as

\[ \gamma[n] = \frac{(1 - \alpha)\gamma_{SR}[n]\gamma_{RD}[n]}{(\alpha \gamma_{SR}[n] + 1)\gamma_{RD}[n] + \frac{P_{tx}[n]}{\sigma^2_R\beta^2}} \]

where \( \alpha = 1 - |A_R[0]|^2 = \sum_{k=1}^{N_c-1} |A_R[k]|^2 \) represents ICI power and SNRs are \( \gamma_{SR}[n] = P_S[n]|H_{SR}[n]|^2/\sigma^2_R \), \( \gamma_{RD}[n] = P_{tx}[n]|H_{RD}[n]|^2/\sigma^2_D \)
Non-ideal Oscillators in the Repeater

- In the following: Comparison of two different repeater designs

(a) Two separate oscillators

- downconversion:
  \[ a_{\text{RX}}(t) = e^{-j(2\pi f_c t - \theta_{\text{RX}}(t))} \]
- upconversion:
  \[ a_{\text{TX}}(t) = e^{j(2\pi f_c t + \theta_{\text{TX}}(t))} \]

(b) Reusing single oscillator

- downconversion:
  \[ a_{\text{RX}}(t) = e^{-j(2\pi f_c t - \theta_{\text{RX}}(t))} \]
- upconversion:
  \[ a_{\text{TX}}(t) = a^*_{\text{RX}}(t) = e^{j(2\pi f_c t - \theta_{\text{TX}}(t))} \]

- The total phase distortion caused by phase noise and repeater processing delay \( \tau \) can be captured as phasor process

\[ a_{\text{R}}(t) = a_{\text{RX}}(t - \tau) \cdot a_{\text{TX}}(t) \]
Wiener Phase Noise

• The phase noise of free-running oscillators can be modelled accurately as a Wiener process, i.e., standard Brownian motion or “random walk with Gaussian steps”:

\[
\theta_{\text{rx}}(t_0) - \theta_{\text{rx}}(t_0 - t) \sim \mathcal{N}(0, c_{\text{rx}} \cdot |t|)
\]
\[
\theta_{\text{tx}}(t_0) - \theta_{\text{tx}}(t_0 - t) \sim \mathcal{N}(0, c_{\text{tx}} \cdot |t|)
\]

• (In)dependence of rx and tx sides due to the repeater design
  ▶ Two separate oscillators: \( \theta_{\text{tx}}(t) \) is independent of \( \theta_{\text{rx}}(t) \)
  ▶ Reusing single oscillator: \( \theta_{\text{tx}}(t) = -\theta_{\text{rx}}(t) \)

• The quality of the oscillator is parametrized by \( f_{3\text{dB}} \) which defines the 3dB bandwidth of the oscillator power spectral density (PSD)
  ▶ When using two oscillators, they are assumed to be of similar quality in this study: \( c = c_{\text{rx}} = c_{\text{tx}} = 4\pi f_{3\text{dB}} \)
Spectral Spreading due to Phase Noise
Example (1): Long Processing Delay

(a) Two separate oscillators

\[ \theta_{\text{rx}}(t - \tau) \]

\[ \theta_{\text{tx}}(t) \]

\[ \angle a_R(t) - 2\pi f_c \tau \]

(b) Reusing single oscillator
Example (2): Short Processing Delay

(a) Two separate oscillators

\[ \theta_{rx}(t - \tau) \]

\[ \theta_{tx}(t) \]

\[ \angle a_R(t) - 2\pi f_c \tau \]

(b) Reusing single oscillator
PSD of Repeater Phasor Process

- Total phase distortion caused by the repeater
  \[ a_R(t) = a_{rx}(t - \tau) \cdot a_{tx}(t) = \begin{cases} e^{j(2\pi f_c \tau + \theta_{rx}(t-\tau) + \theta_{tx}(t))}, & \text{two oscillators} \\ e^{j(2\pi f_c \tau + \theta_{rx}(t-\tau) - \theta_{rx}(t))}, & \text{one oscillator} \end{cases} \]

- ICI is realized through \( A_R[k] \) which represents instantaneous spectral spreading for each OFDM symbol

- Standard steps for calculating power spectral density (PSD): first \( R(t) = \mathcal{E}\{a_R(t_0)a_R^*(t_0 - t)\} \) then \( S(f) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(t)e^{-j2\pi ft} dt \)
  - PSD is related to \( \mathcal{E}\{|A_R[k]|^2\} \), i.e., expected ICI power
  - In the ideal case \( a_R(t) = e^{j2\pi f_c \tau} \) yielding \( S_0(f) = \delta(f) \)

(a) Two separate oscillators
\[
S_2(f) = \frac{1}{\pi} \cdot \frac{c}{c^2 + (2\pi f)^2}
\]

(b) Reusing single oscillator
\[
S_1(f) = e^{-c\tau} S_0(f) + \tilde{S}_1(f) S_2(f)
\]

where
\[
\tilde{S}_1(f) = 1 - e^{-c\tau} \left( \cos(2\pi f \tau) + c\tau \text{sinc}(2\pi f \tau) \right)
\]
Numerical Results (1)

- On the right: PSD when reusing single oscillator and $f_{3\text{dB}} = 100\text{Hz}$
- Extreme cases for processing delay:
  $$S_1(f) \rightarrow \begin{cases} 
  S_0(f), & \text{when } \tau \to 0 \\
  S_2(f), & \text{when } \tau \to \infty 
  \end{cases}$$
  (cf. OFDM sample and symbol duration)

- Except for the impulse at the zero frequency (not visible in the above figure), the PSD is approximately flat when $f < \frac{1}{4\tau}$
- When $f > \frac{1}{4\tau}$, the PSD decays 20dB per decade
Numerical Results (2)

- On the right: PSD vs. processing delay when reusing single oscillator and $f_{3dB} = 100Hz$
  - at 1kHz: Flat PSD
  - at 1MHz: PSD decays 20 dB per decade
  (cf. subcarrier spacing and bandwidth in OFDM)

- The PSD oscillates less when the processing delay increases and becomes smooth when $\tau > \frac{1}{4f_{3dB}}$ (≈ $\infty$)
  - However, OFDM symbols are typically shorter than that
Transmission Rate Analysis
Distribution of ICI Power

- When reusing a single oscillator, time-domain phase distortion \( \angle a_R(t) \) can be seen as colored Gaussian noise.
- And \( A_R[k] \) represents \( a_R(t) \) in frequency domain after sampling.
- Using Taylor series expansion, ICI power \( \alpha = \sum_{k=1}^{N_c-1} |A_R[k]|^2 \) becomes a sum of correlated gamma random variables.

\[ \triangleright \text{Coefficients } \lambda_k \text{ depend on } f_{3\text{dB}} \text{ and } \tau \text{ via a covariance matrix!} \]

- Finally, the probability density function (PDF) of \( \alpha \) can be expressed as a weighted sum of gamma PDFs:

\[
p(\alpha) = \kappa \sum_{k=0}^{\infty} \zeta_k p_k(\alpha) \quad \text{where} \quad p_k(\alpha) = \frac{\alpha^{N_c-1+k-1} e^{-\alpha \lambda_1}}{\lambda_1^{N_c-1+k} \Gamma \left( \frac{N_c-1}{2} + k \right)}
\]

\[ \triangleright \kappa = \prod_{n=1}^{N_c-1} \sqrt{\frac{\lambda_1}{\lambda_n}} \quad \text{(probability mass normalization)} \]

\[ \triangleright \zeta_0 = 1 \quad \text{and} \quad \zeta_{k+1} = \frac{1/2}{k+1} \sum_{i=1}^{k+1} \sum_{j=1}^{N_c-1} \left( 1 - \frac{\lambda_1}{\lambda_j} \right)^i \zeta_{k+1-i} \]
**Average Transmission Rate**

- Repeater gain \( \beta^2 = \frac{P_{tx}[n]/\sigma_R^2}{(\gamma_{SR}[n] + 1)} \) transforms SINR to
  \[
  \gamma[n] = \frac{(1 - \alpha)\gamma_{SR}[n]\gamma_{RD}[n]}{\gamma_{SR}[n] + (\alpha \gamma_{SR}[n] + 1)\gamma_{RD}[n] + 1}
  \]

- Instantaneous transmission rate is given by
  \[
  C[n] = \log_2(1+\gamma[n]) = \log_2 \left( \frac{\gamma_{SR}[n]\gamma_{RD}[n] + \gamma_{SR}[n] + \gamma_{RD}[n] + 1}{(\alpha \gamma_{SR}[n]\gamma_{RD}[n] + \gamma_{SR}[n] + \gamma_{RD}[n] + 1)} \right)
  \]

- Using \( p(\alpha) \), average transmission rate can be calculated as
  \[
  \bar{C}[n] = \mathcal{E}\{C[n]\} = \log_2 \left( 1 + \frac{\gamma_{SR}[n]\gamma_{RD}[n]}{\gamma_{SR}[n] + \gamma_{RD}[n] + 1} \right) - \kappa \sum_{k=0}^{\infty} \zeta_k I_k
  \]

  where
  \[
  I_k = \int_0^{\infty} \log_2 \left( 1 + \frac{\gamma_{SR}[n]\gamma_{RD}[n]}{\gamma_{SR}[n] + \gamma_{RD}[n] + 1} \alpha \right) p_k(\alpha) \, d\alpha
  \]

  \( I_k \) can be solved in a closed form using Meijer’s G-function
  (or generalized hypergeometric and incomplete gamma functions)
Numerical Results (3)

- OFDM parameters for a DVB-T/H-like system:
  \( N_c = 8192 \) subcarriers and 8MHz bandwidth
  – sample duration: 0.11\( \mu \)s
  – FFT duration: 896\( \mu \)s
  – subcarrier spacing: 1.1kHz

- Oscillators: \( f_{3dB} = 100\text{Hz} \)

- When reusing a single oscillator, transmission rate degradation can be minimized by decreasing the processing delay
  ▶ Implementation with two separate oscillators means \( \tau \rightarrow \infty \)
Numerical Results (4)

- OFDM parameters for a DVB-T/H-like system:
  \[ N_c = 8192 \text{ subcarriers} \]
  \[ \text{and } 8\text{MHz bandwidth} \]
  \[ \text{– sample duration: } 0.11\mu s \]
  \[ \text{– FFT duration: } 896\mu s \]
  \[ \text{– subcarrier spacing: } 1.1\text{kHz} \]
- Oscillators: \( f_{3\text{dB}} = 100\text{Hz} \)

- If the processing delay is a few tens of OFDM samples or shorter, the transmit-side noise reverts the effect of receive-side noise
  ▶ The delay needs to be shorter than the cyclic prefix anyway
Conclusion

- Target: To understand the effect of spectral spreading on a full-duplex OFDM repeater link due to imperfect oscillator(s)
  - Phase noise causes inter-carrier interference (ICI)
- Comparison of two different repeater designs
  1. Using a single oscillator signal for down- and upconversion: *Processing delay becomes a key factor for spectral spreading!*
  2. Separate oscillators for down- and upconversion
- Analysis and numerical results at three abstraction levels
  1. Time-domain phase noise realizations vs. processing delay
  2. Power spectral density of repeater’s phase distortion process
  3. Distribution of ICI power, and average transmission rate
- The transmit-side phase noise *can* partially revert the effect of receive-side distortion when processing delay is short enough