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# Residual Self-Interference in Full-Duplex MIMO Relays After Null-Space Projection and Cancellation

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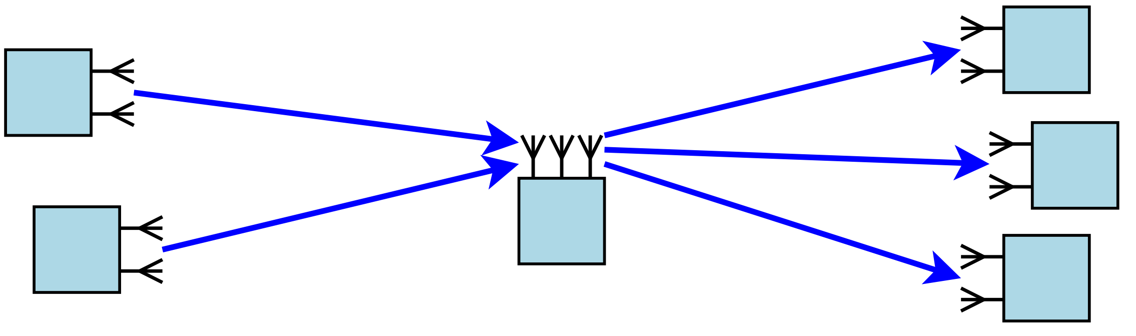
# Introduction

# Background

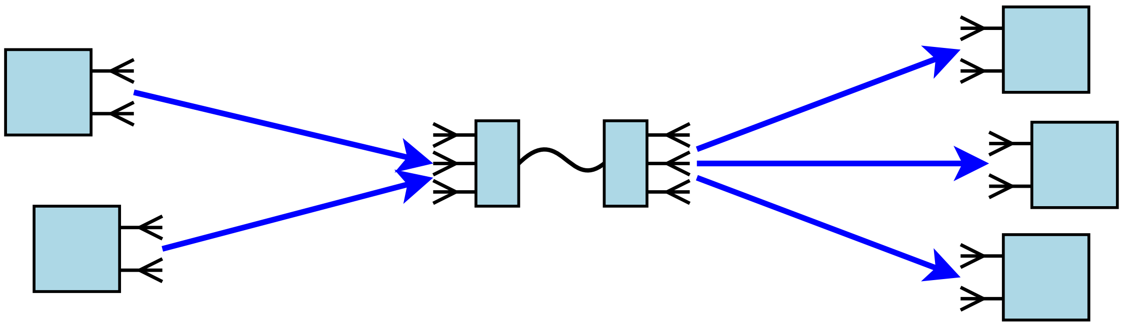
- Two-hop MIMO relay links have been a hot research topic recently



- Most of the literature assumes half-duplex relaying mode
  - Different time slots or frequency bands for relay rx and tx
  - Disadvantage: Loss in spectral efficiency
  - Inevitable choice for relays with single antenna array: Weaker desired signal will be drowned by strong self-interference

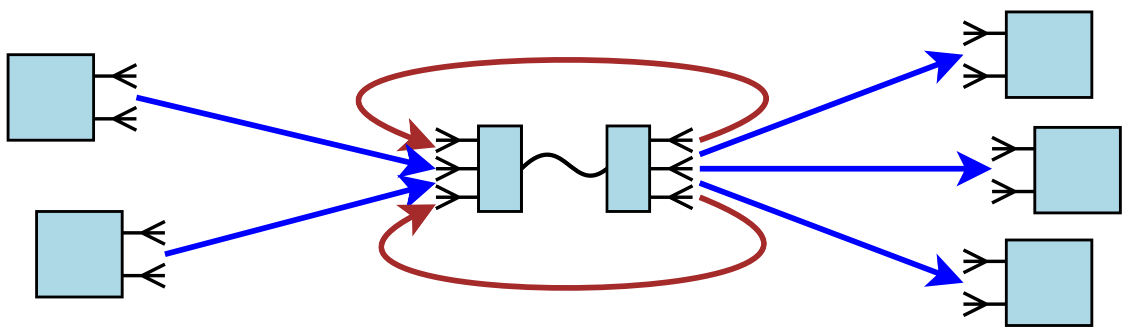
# Goal: Improved Spectral Efficiency

- MIMO relay with separated receive and transmit antenna arrays



# Goal: Improved Spectral Efficiency

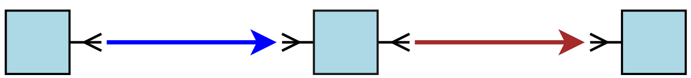
- MIMO relay with separated receive and transmit antenna arrays



- Now it is viable to choose the full-duplex mode, and avoid the loss of spectral efficiency which is inherent for the half-duplex mode
  - ▷ Natural isolation facilitates the usage of signal processing techniques for mitigating the self-interference
- Practical problem: There will be still residual self-interference

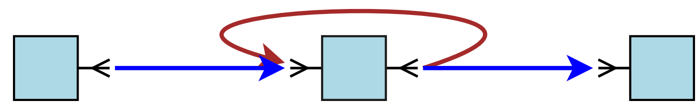
# Motivation from Our SISO Relay Studies (1)

- The choice between half-duplex and full-duplex relaying represents a fundamental rate–interference tradeoff
  - ▷ Half-duplex relay link:
  - ▷ Full-duplex relay link:



Reduced symbol rate due to two allocated channels

$$\mathcal{R}_{\mathrm{HD}} = \frac{1}{2}\log_2\left(1+\frac{P_{\mathrm{S}}}{P_{\mathrm{N}}}\right)$$



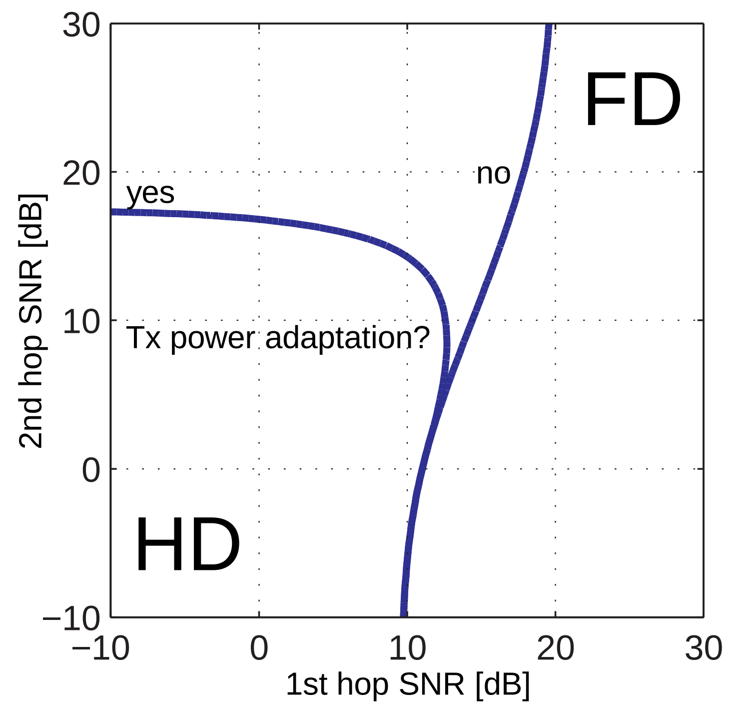
Residual self-interference even after mitigation

$$\mathcal{R}_{\mathrm{FD}} = \log_2\left(1+\frac{P_{\mathrm{S}}}{P_{\mathrm{I}}+P_{\mathrm{N}}}\right)$$

- We have analyzed various aspects of this tradeoff earlier in the case of two-hop SISO relays [WCNC'09, SPAWC'09, PIMRC'10]

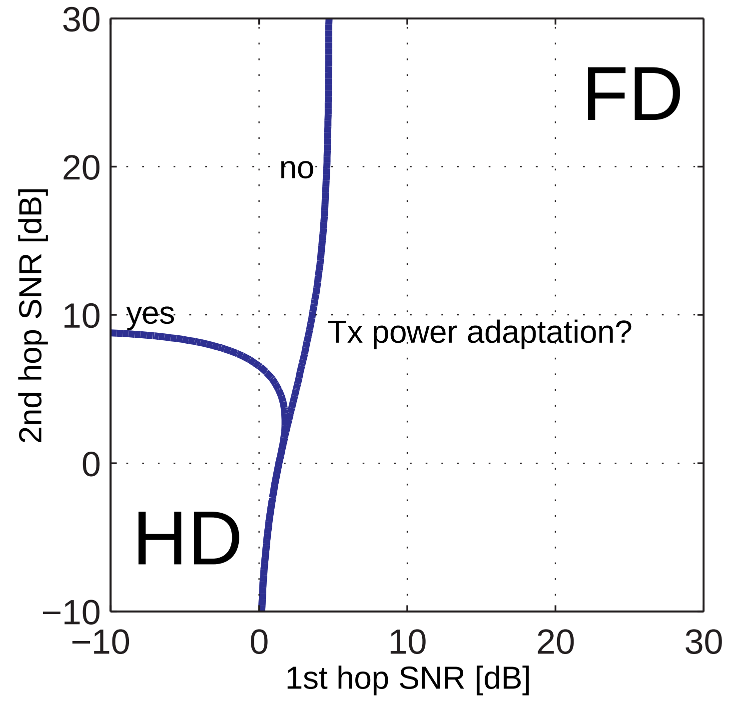
# Motivation from Our SISO Relay Studies (2)

- Half duplex (HD) vs. full duplex (FD)
  - ▷ Which mode gives the best spectral efficiency?
    - — More details in [SPAWC'09]
  - ▷ Fig.: Decision boundaries for $10$dB interference to noise ratio



# Motivation from Our SISO Relay Studies (2)

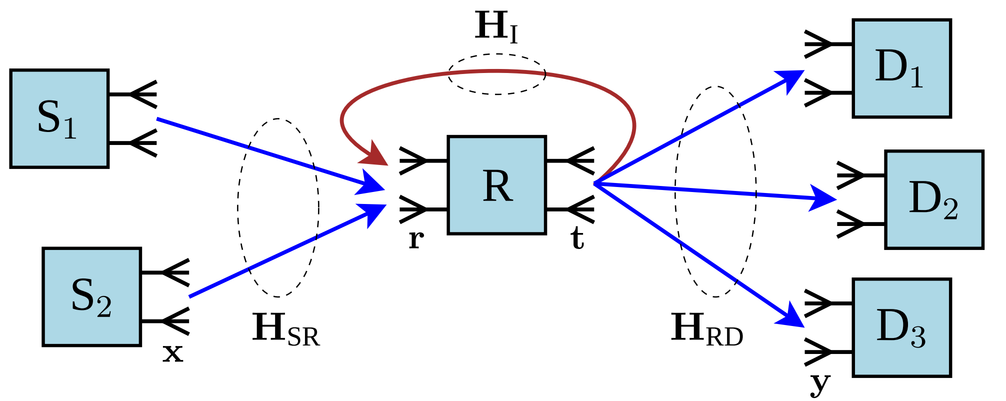
- Half duplex (HD) vs. full duplex (FD)
  - ▷ Which mode gives the best spectral efficiency?
    - — More details in [SPAWC'09]
  - ▷ Fig.: Decision boundaries for $3$dB interference to noise ratio




- This tradeoff can be extrapolated to the case of MIMO relays
  $\Rightarrow$ Development of mitigation schemes to improve isolation
  $\Rightarrow$ Evaluation of self-interference remaining in practice

# System Model

# Signal Model




- Two-hop transmission through a full-duplex MIMO relay:

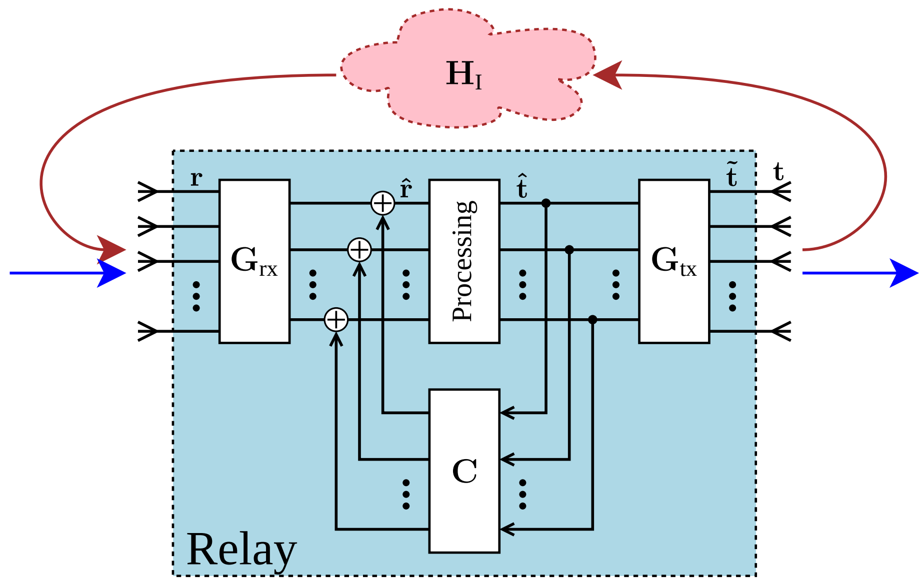
$$\begin{cases} \text{Sources transmit:} & \mathbf{x} & & \in \mathbb{C}^{N_\mathrm{S}\times 1} \\ \text{Relay receives:} & \mathbf{r} = \mathbf{H}_\mathrm{SR}\mathbf{x} + \mathbf{H}_\mathrm{I}\mathbf{t} & + \mathbf{n}_\mathrm{R} & \in \mathbb{C}^{N_\mathrm{rx}\times 1} \\ \text{Relay transmits:} & \mathbf{t} & & \in \mathbb{C}^{N_\mathrm{tx}\times 1} \\ \text{Destinations receive:} & \mathbf{y} = \mathbf{H}_\mathrm{RD}\mathbf{t} + \mathbf{H}_\mathrm{SD}\mathbf{x} & + \mathbf{n}_\mathrm{D} & \in \mathbb{C}^{N_\mathrm{D}\times 1} \end{cases}$$

# Side Information for Transparent Mitigation

- We concentrate on *transparent* mitigation techniques
  - ▷ $\mathbf{t}$ is known by design and $\mathbf{H}_{\mathrm{I}}$ can be estimated in the relay
  - ▷ Feedback overhead is avoided
- However, the side information can still be imperfect in practice
  - ▷ Mitigation may use only the estimates $\tilde{\mathbf{H}}_{\mathrm{I}}$, $\tilde{\mathbf{t}}$ of $\mathbf{H}_{\mathrm{I}}$, $\mathbf{t}$
    - – Channel estimation error: $\boldsymbol{\Delta}\tilde{\mathbf{H}}_{\mathrm{I}} = \mathbf{H}_{\mathrm{I}} - \tilde{\mathbf{H}}_{\mathrm{I}}$
    - – Transmit signal noise: $\boldsymbol{\Delta}\tilde{\mathbf{t}} = \mathbf{t} - \tilde{\mathbf{t}}$
- The noise in the side information is modeled as i.i.d. Gaussian
  - ▷ $\epsilon_{\mathbf{H}}^2 = \mathcal{E}\{\|\boldsymbol{\Delta}\tilde{\mathbf{H}}_{\mathrm{I}}\|_F^2\}/\mathcal{E}\{\|\mathbf{H}_{\mathrm{I}}\|_F^2\}$
  - ▷ $\epsilon_{\mathbf{t}}^2 = \mathcal{E}\{\|\boldsymbol{\Delta}\tilde{\mathbf{t}}\|_2^2\}/\mathcal{E}\{\|\mathbf{t}\|_2^2\}$

# Mitigation of Self-Interference

# Mitigation of Self-Interference

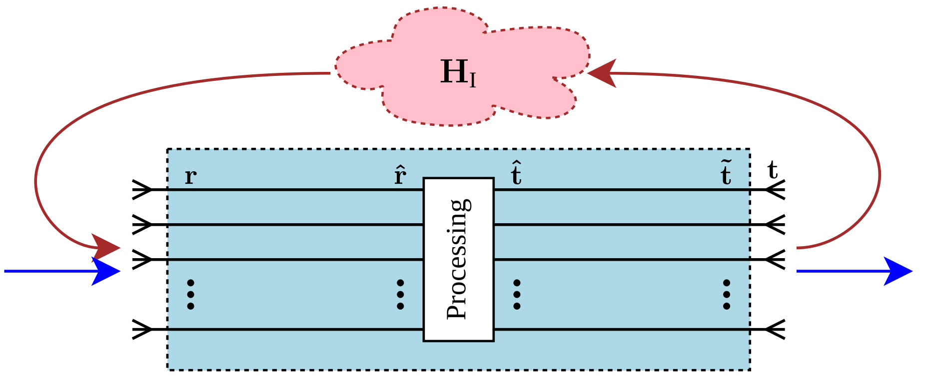



- Cancellation filter $\mathbf{C}$, spatial suppression filters $\mathbf{G}_{\mathrm{rx}}$ and $\mathbf{G}_{\mathrm{tx}}$
- Transforms the relay with $N_{\mathrm{rx}} \times N_{\mathrm{tx}}$ antennas to an "interference-free" relay with $\hat{N}_{\mathrm{rx}} \times \hat{N}_{\mathrm{tx}}$ streams:

$$\begin{cases} \hat{\mathbf{r}} = \mathbf{G}_{\mathrm{rx}}\mathbf{r} + \mathbf{C}\hat{\mathbf{t}} \in \mathbb{C}^{\hat{N}_{\mathrm{rx}} \times 1} \\ \mathbf{t} = \mathbf{G}_{\mathrm{tx}}\hat{\mathbf{t}} + \mathbf{\Delta}\tilde{\mathbf{t}} \in \mathbb{C}^{N_{\mathrm{tx}} \times 1} \end{cases}$$

- Any protocol can be used for generating $\hat{\mathbf{t}}$ based on $\hat{\mathbf{r}}$

# Natural Isolation

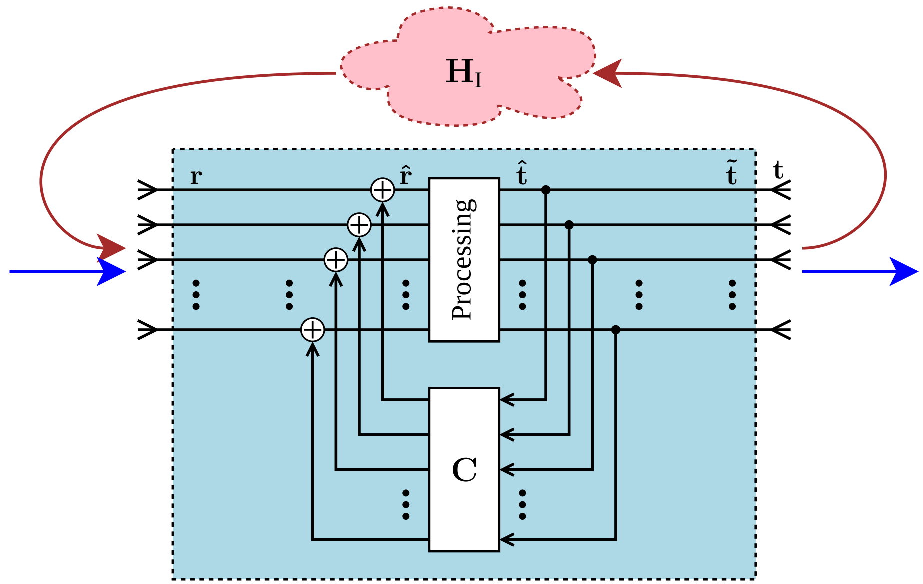



- The reference case without mitigation: $\mathbf{C} = \mathbf{0}$, $\mathbf{G}_\mathrm{rx} = \mathbf{I}$, $\mathbf{G}_\mathrm{tx} = \mathbf{I}$
- Self-interference power becomes

$$P_\mathrm{I} = \mathcal{E}\{\|\mathbf{H}_\mathrm{I}\mathbf{t}\|_2^2\} = (1+\epsilon_{\mathbf{t}}^2)\mathrm{tr}\{\mathbf{H}_\mathrm{I}\mathbf{R}_{\hat{\mathbf{t}}}\mathbf{H}_\mathrm{I}^H\} = N_\mathrm{rx}(1+\epsilon_{\mathbf{t}}^2)P_\mathrm{tx}$$

  - ▷ We assume that $\mathbf{R}_{\hat{\mathbf{t}}} = \mathcal{E}\{\hat{\mathbf{t}}\hat{\mathbf{t}}^H\} = \frac{P_\mathrm{tx}}{N_\mathrm{tx}}\mathbf{I}$
    and normalize the channel gain as $\|\mathbf{H}_\mathrm{I}\|_F^2 = N_\mathrm{rx}N_\mathrm{tx}$

# Time-Domain Cancellation (TDC)

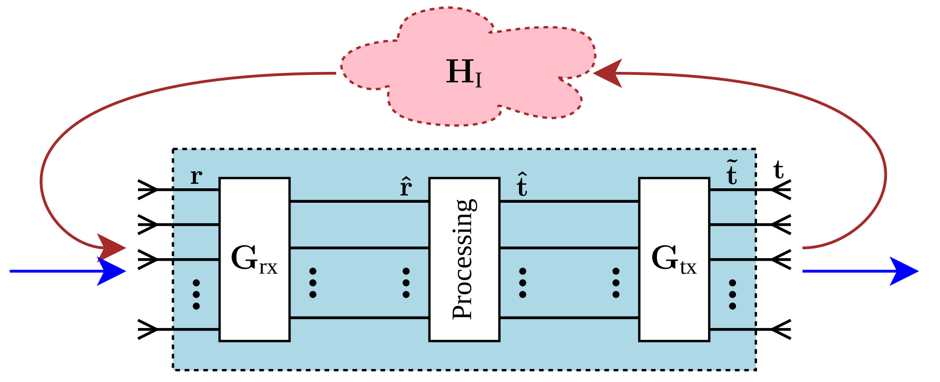



- No spatial filters: $\mathbf{G}_\mathrm{rx} = \mathbf{I}$, $\mathbf{G}_\mathrm{tx} = \mathbf{I}$
- Subtract an estimated interference signal from the input signal

$$\mathbf{C} = -\tilde{\mathbf{H}}_\mathrm{I}$$

  - Cancellation is imperfect because $\tilde{\mathbf{H}}_\mathrm{I} \neq \mathbf{H}_\mathrm{I}$ in practice
  - The unknown transmit signal noise $\mathbf{\Delta}\tilde{\mathbf{t}}$ cannot be cancelled

# Null-Space Projection (NSP)

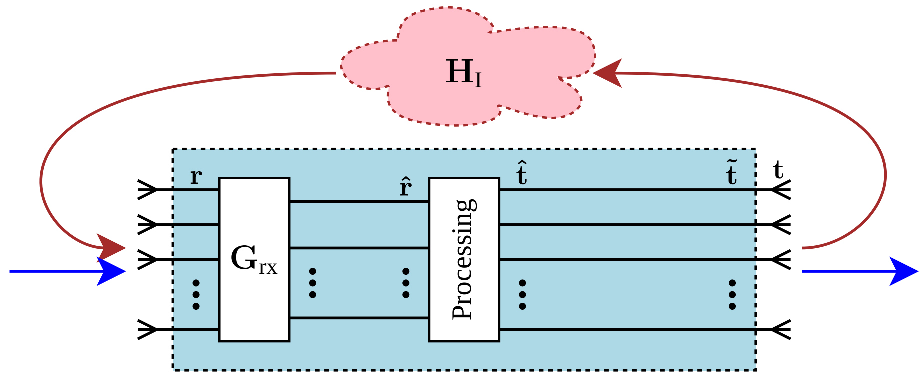



- No cancellation filter: $\mathbf{C} = \mathbf{0}$
- Choose $\mathbf{G}_\mathrm{rx} = \sqrt{N_\mathrm{rx}/\hat{N}_\mathrm{rx}}\mathbf{S}_\mathrm{rx}^T\tilde{\mathbf{U}}^H$, $\mathbf{G}_\mathrm{tx} = \sqrt{N_\mathrm{tx}/\hat{N}_\mathrm{tx}}\tilde{\mathbf{V}}\mathbf{S}_\mathrm{tx}$ such that

$$\mathbf{G}_\mathrm{rx}\tilde{\mathbf{H}}_\mathrm{I}\mathbf{G}_\mathrm{tx} = \mathbf{G}_\mathrm{rx}\tilde{\mathbf{U}}\tilde{\mathbf{\Sigma}}\tilde{\mathbf{V}}^H\mathbf{G}_\mathrm{tx} = \sqrt{\frac{N_\mathrm{rx}N_\mathrm{tx}}{\hat{N}_\mathrm{rx}\hat{N}_\mathrm{tx}}}\mathbf{S}_\mathrm{rx}^T\tilde{\mathbf{\Sigma}}\mathbf{S}_\mathrm{tx} = \mathbf{0}$$

  - ▷ $\hat{N}_\mathrm{rx} + \hat{N}_\mathrm{tx} = \max\{N_\mathrm{rx}, N_\mathrm{tx}\}$ when $\tilde{\mathbf{H}}_\mathrm{I}$ is of full rank
  - ▷ Example selection matrices: $\mathbf{S}_\mathrm{rx}^T = [\mathbf{0}|\mathbf{I}]$ and $\mathbf{S}_\mathrm{tx} = [\mathbf{I}|\mathbf{0}]^T$ when $N_\mathrm{rx} \geq N_\mathrm{tx}$

# NSP for Suppressing Transmit Signal Noise




- Transmit filter cannot suppress the transmit signal noise: $\mathbf{G}_\mathrm{tx} = \mathbf{I}$
- Choose $\mathbf{G}_\mathrm{rx} = \sqrt{N_\mathrm{rx}/\hat{N}_\mathrm{rx}} \mathbf{S}_\mathrm{rx}^T \tilde{\mathbf{U}}^H$ such that

$$\mathbf{G}_\mathrm{rx}\tilde{\mathbf{H}}_\mathrm{I} = \mathbf{G}_\mathrm{rx}\tilde{\mathbf{U}}\tilde{\mathbf{\Sigma}}\tilde{\mathbf{V}}^H = \sqrt{\frac{N_\mathrm{rx}}{\hat{N}_\mathrm{rx}}} \mathbf{S}_\mathrm{rx}^T \tilde{\mathbf{\Sigma}}\tilde{\mathbf{V}}^H = \mathbf{0}$$

  which also implies $\mathbf{G}_\mathrm{rx}\tilde{\mathbf{H}}_\mathrm{I}\mathbf{G}_\mathrm{tx} = \mathbf{0}$

  - ▷ Unique choice for the row selection matrix: $\mathbf{S}_\mathrm{rx}^T = [\mathbf{0}|\mathbf{I}]$
  - ▷ $\hat{N}_\mathrm{rx} = N_\mathrm{rx} - N_\mathrm{tx}$ and $N_\mathrm{rx} > N_\mathrm{tx}$ when $\tilde{\mathbf{H}}_\mathrm{I}$ is of full rank

# Residual Self-Interference

# Isolation Improvement

- Additional isolation from mitigation w.r.t. natural isolation:

$$\Delta P_{\mathrm{I}} = \frac{P_{\mathrm{I}}\big|_{\mathrm{natural}}}{P_{\mathrm{I}}\big|_{\mathrm{mitigation}}} = \frac{N_{\mathrm{rx}}(1+\epsilon_{\mathbf{t}}^2)P_{\mathrm{tx}}}{P_{\mathrm{I}}\big|_{\mathrm{mitigation}}}$$

  which is a random variable due to noisy side information

- The residual self-interference can be analyzed in terms of
  - cumulative distribution function: $F_{\Delta P_{\mathrm{I}}}(\cdot)$
  - average improvement of isolation: $\mathcal{E}\{\Delta P_{\mathrm{I}}\}$

# Some Statistical Tools

- Let $X$ be a gamma random variable with shape $k$ and scale $\theta$, i.e., $X \sim \mathcal{G}(k, \theta)$
  - ▷ Cumulative distribution function: $F_X(x) = 1 - \frac{\Gamma(k, x/\theta)}{\Gamma(k)}$ for $x \geq 0$
- We will see random variables of the form

$$Y = \frac{a}{X + b}, \quad a \geq 0,\, b \geq 0$$

  - ▷ CDF: $F_Y(y) = 1 - F_X\left(\frac{a}{y} - b\right)$ for $0 \leq y \leq \frac{a}{b}$
  - ▷ Average: $\mathcal{E}\{Y\} = \frac{a}{\theta} \exp\left(\frac{b}{\theta}\right) E_k\left(\frac{b}{\theta}\right)$
    - – $E_k(\cdot)$ is the exponential integral

# Time-Domain Cancellation (TDC)

- Residual self-interference power is

$$P_{\mathrm{I}} = \mathcal{E}\{\|\boldsymbol{\Delta}\tilde{\mathbf{H}}_{\mathrm{I}}\hat{\mathbf{t}} + \mathbf{H}_{\mathrm{I}}\boldsymbol{\Delta}\tilde{\mathbf{t}}\|_2^2\} = N_{\mathrm{rx}}\left(\frac{\|\boldsymbol{\Delta}\tilde{\mathbf{H}}_{\mathrm{I}}\|_F^2}{N_{\mathrm{rx}}N_{\mathrm{tx}}} + \epsilon_{\mathbf{t}}^2\right)P_{\mathrm{tx}}$$

- Thus, isolation improvement becomes

$$\Delta P_{\mathrm{I}} = \frac{1+\epsilon_{\mathbf{t}}^2}{X+\epsilon_{\mathbf{t}}^2}$$

where $X \sim \mathcal{G}\left(N_{\mathrm{rx}}N_{\mathrm{tx}}, \frac{\epsilon_{\mathbf{H}}^2}{N_{\mathrm{rx}}N_{\mathrm{tx}}}\right)$

# Null-Space Projection (NSP)

- Residual self-interference power is

$$P_{\mathrm{I}} = \mathcal{E}\{\|\mathbf{G}_{\mathrm{rx}}\boldsymbol{\Delta}\tilde{\mathbf{H}}_{\mathrm{I}}\mathbf{G}_{\mathrm{tx}}\hat{\mathbf{t}} + \mathbf{G}_{\mathrm{rx}}\mathbf{H}_{\mathrm{I}}\boldsymbol{\Delta}\tilde{\mathbf{t}}\|_2^2\}$$

$$= N_{\mathrm{rx}} \left( \frac{\|\mathbf{S}_{\mathrm{rx}}^T\tilde{\mathbf{U}}^H\boldsymbol{\Delta}\tilde{\mathbf{H}}_{\mathrm{I}}\tilde{\mathbf{V}}\mathbf{S}_{\mathrm{tx}}\|_F^2}{\hat{N}_{\mathrm{rx}}\hat{N}_{\mathrm{tx}}} + \epsilon_{\mathbf{t}}^2 \frac{\|\mathbf{S}_{\mathrm{rx}}^T\tilde{\mathbf{U}}^H\mathbf{H}_{\mathrm{I}}\|_F^2}{\hat{N}_{\mathrm{rx}}N_{\mathrm{tx}}} \right) P_{\mathrm{tx}}$$

where $\|\mathbf{S}_{\mathrm{rx}}^T\tilde{\mathbf{U}}^H\mathbf{H}_{\mathrm{I}}\|_F^2 \approx \|\mathbf{S}_{\mathrm{rx}}^T\mathbf{U}^H\mathbf{H}_{\mathrm{I}}\|_F^2$ (a constant)

- Thus, isolation improvement becomes

$$\Delta P_{\mathrm{I}} \approx \frac{1+\epsilon_{\mathbf{t}}^2}{X+\epsilon_{\mathbf{t}}^2 A}$$

where $X \sim \mathcal{G}\left(\hat{N}_{\mathrm{rx}}\hat{N}_{\mathrm{tx}}, \frac{\epsilon_{\mathbf{H}}^2}{\hat{N}_{\mathrm{rx}}\hat{N}_{\mathrm{tx}}}\right)$ and $A = \frac{\|\mathbf{S}_{\mathrm{rx}}^T\mathbf{U}^H\mathbf{H}_{\mathrm{I}}\|_F^2}{\hat{N}_{\mathrm{rx}}N_{\mathrm{tx}}}$

# NSP for Suppressing Transmit Signal Noise

- Residual self-interference power is

$$P_{\mathrm{I}} = \mathcal{E}\{\|\mathbf{G}_{\mathrm{rx}}\boldsymbol{\Delta}\tilde{\mathbf{H}}_{\mathrm{I}}\hat{\mathbf{t}} + \mathbf{G}_{\mathrm{rx}}\boldsymbol{\Delta}\tilde{\mathbf{H}}_{\mathrm{I}}\boldsymbol{\Delta}\tilde{\mathbf{t}}\|_2^2\}$$
$$= N_{\mathrm{rx}}(1+\epsilon_{\mathbf{t}}^2)\frac{\|\mathbf{S}_{\mathrm{rx}}^T\tilde{\mathbf{U}}^H\boldsymbol{\Delta}\tilde{\mathbf{H}}_{\mathrm{I}}\|_F^2}{\hat{N}_{\mathrm{rx}}N_{\mathrm{tx}}}P_{\mathrm{tx}}$$

- Thus, isolation improvement becomes

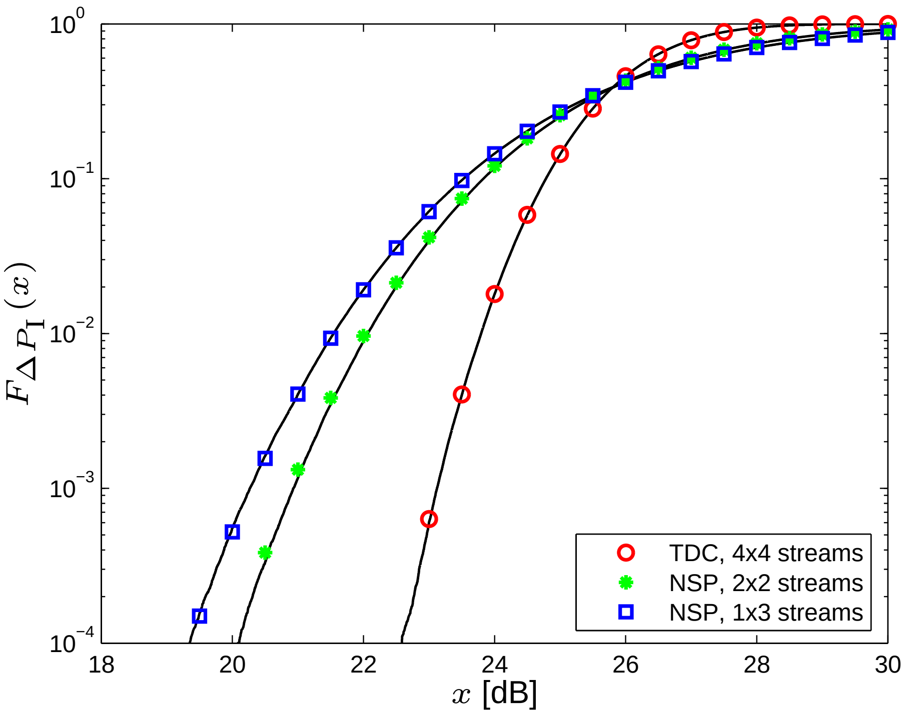
$$\Delta P_{\mathrm{I}} = \frac{1}{X}$$

where $X \sim \mathcal{G}\left(\hat{N}_{\mathrm{rx}}N_{\mathrm{tx}}, \frac{\epsilon_{\mathbf{H}}^2}{\hat{N}_{\mathrm{rx}}N_{\mathrm{tx}}}\right)$

# Numerical Results

# Comparison in Terms of CDF (1)

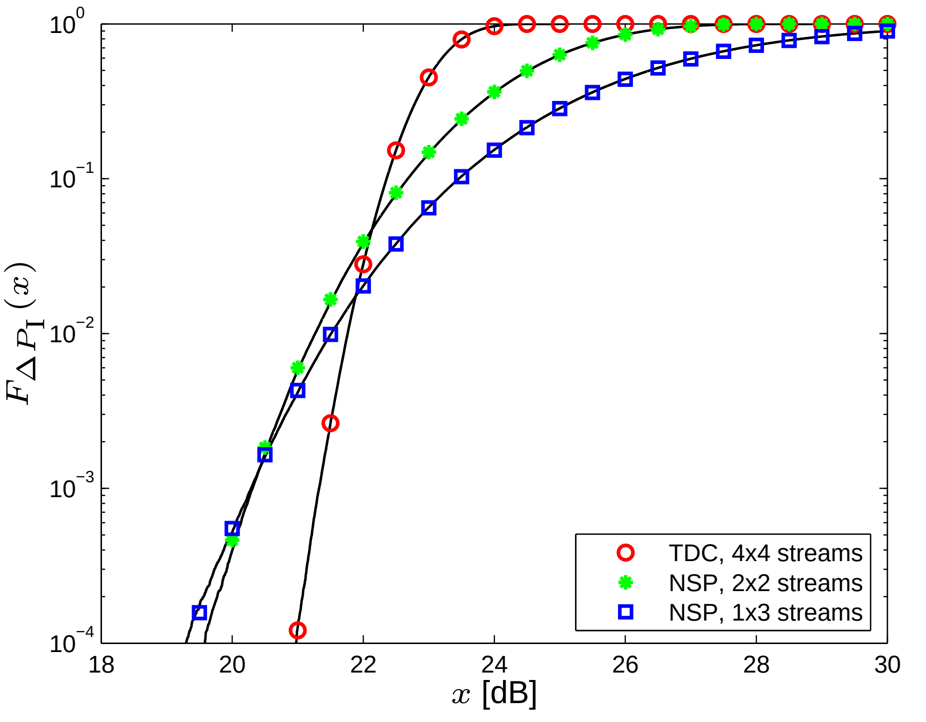
- $N_{\text{rx}} \times N_{\text{tx}} = 4 \times 4$
- $\epsilon_{\mathbf{H}} = 0.05$
- $\epsilon_{\mathbf{t}} = 0$




- TDC: More streams, lower average isolation, lower variance
- NSP: Less streams, higher average isolation, higher variance

# Comparison in Terms of CDF (2)

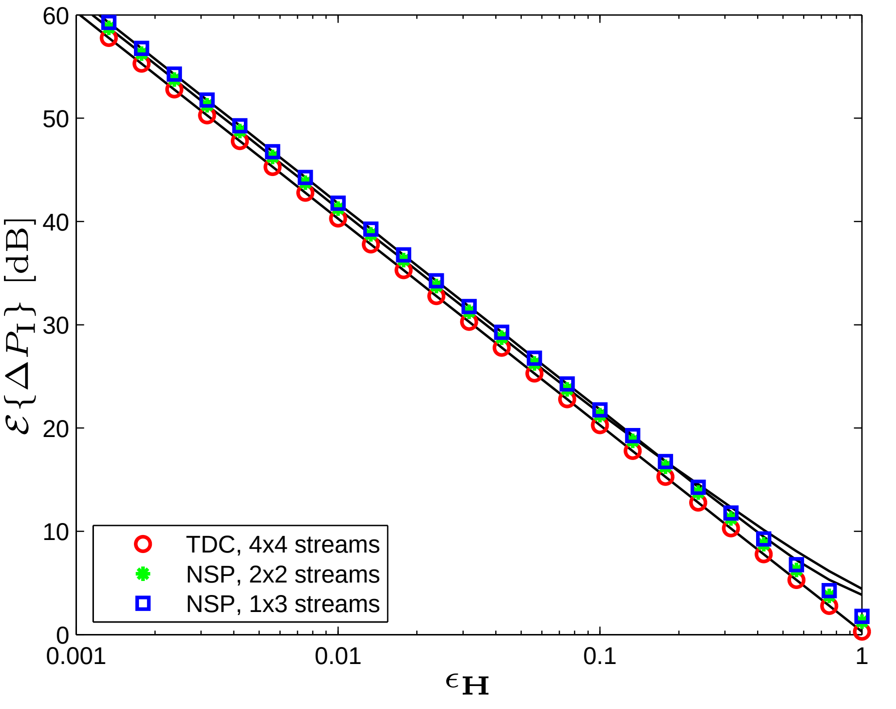
- $N_{\text{rx}} \times N_{\text{tx}} = 4 \times 4$
- $\epsilon_{\mathbf{H}} = 0.05$
- $\epsilon_{\mathbf{t}} = 0.05$




- Transmit signal noise affects more TDC than NSP

# Expected Isolation Improvement (1)

- $N_{\mathrm{rx}} \times N_{\mathrm{tx}} = 4 \times 4$
- $\epsilon_{\mathbf{t}} = 0$




- Isolation improvement decreases linearly as a function of channel estimation error when there is no transmit signal noise

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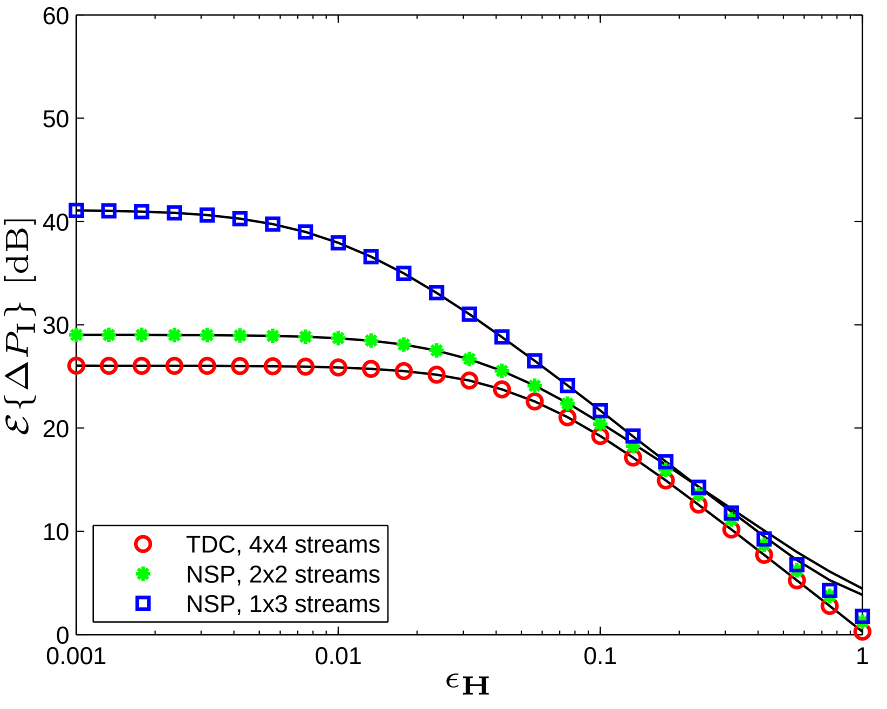
# Expected Isolation Improvement (2)

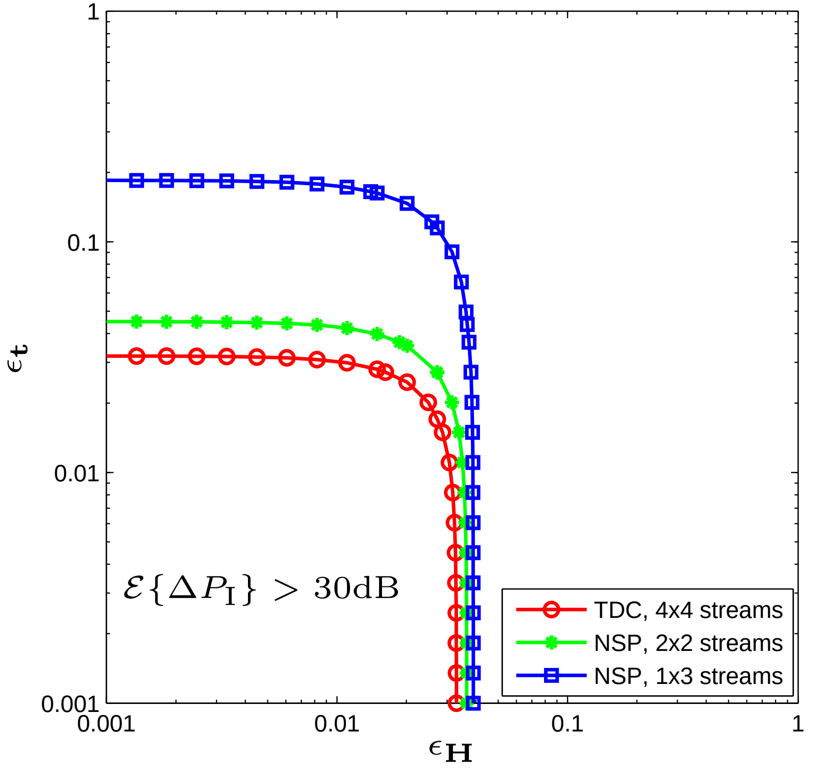
- $N_{\text{rx}} \times N_{\text{tx}} = 4 \times 4$
- $\epsilon_{\mathbf{t}} = 0.05$




- Transmit signal noise limits the maximum isolation improvement

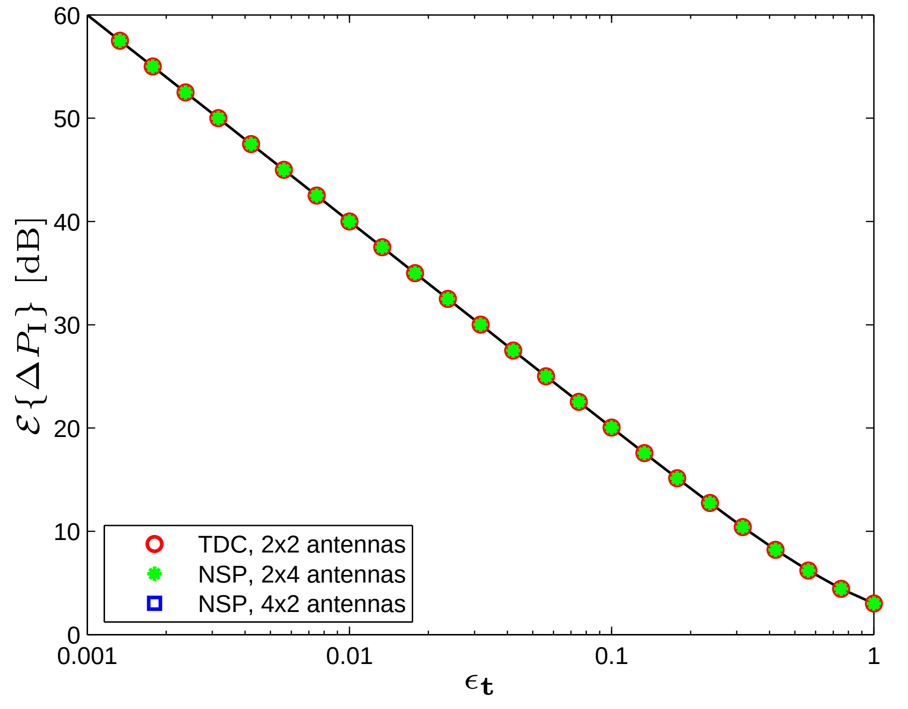
# Robustness to Noisy Side Information

- When is isolation improved by at least $30$dB?
- $N_{\mathrm{rx}} \times N_{\mathrm{tx}} = 4 \times 4$



# NSP for Suppressing Transmit Signal Noise (1)

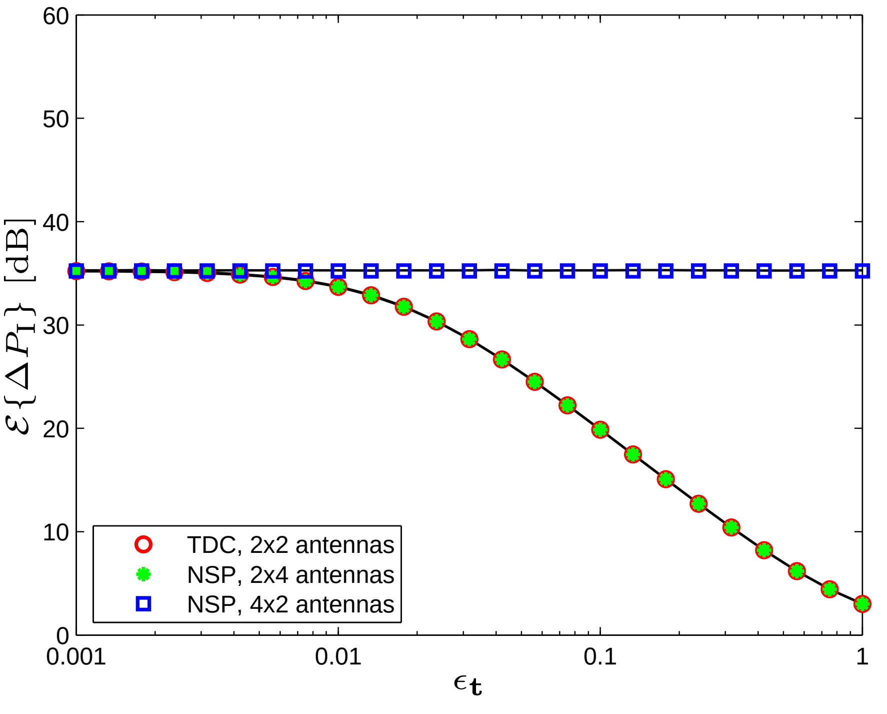
- $\hat{N}_{\mathrm{rx}} \times \hat{N}_{\mathrm{tx}} = 2 \times 2$
- $\epsilon_{\mathbf{H}} = 0$




- Isolation improvement decreases linearly as a function of transmit signal noise level when there is no channel estimation error

# NSP for Suppressing Transmit Signal Noise (2)

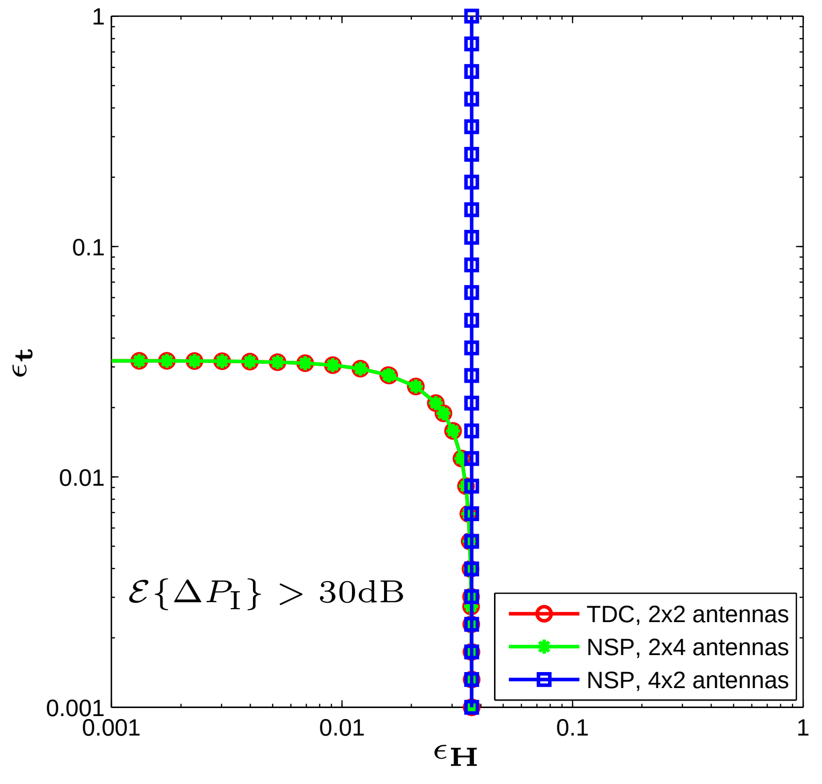
- $\hat{N}_{\mathrm{rx}} \times \hat{N}_{\mathrm{tx}} = 2 \times 2$
- $\epsilon_{\mathbf{H}} = 0.02$




- NSP can be made immune to transmit signal noise

# Robustness to Noisy Side Information

- When is isolation improved by at least $30$dB?
- $\hat{N}_{\mathrm{rx}} \times \hat{N}_{\mathrm{tx}} = 2 \times 2$



# Conclusion

# Conclusion

- Full-duplex relaying has potential to improve spectral efficiency w.r.t. conventional half-duplex relaying
- Main technical problem: self-interference at the relay
  - ▷ Separated antenna arrays in the relay: natural isolation
  - ▷ Signal processing techniques to improve isolation further
    - Time-domain cancellation
    - Spatial-domain suppression: Null-space projection
- Analytical evaluation of self-interference remaining in practice
  - ▷ Side information used in mitigation cannot be perfect
  - ▷ Additional isolation given by mitigation schemes is specified by the noise levels of the side information

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