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BEP Analysis of OFDM Relay Links with Nonlinear Power Amplifiers

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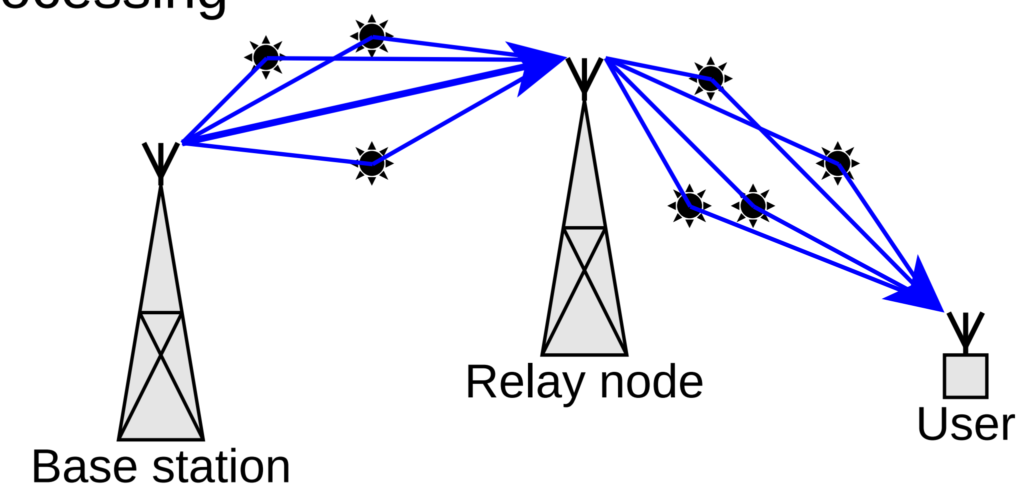
Introduction

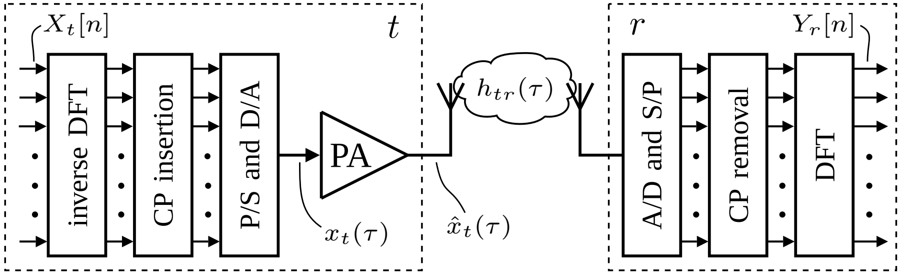
Motivation

- Goal: study the effect of nonlinear distortion in OFDM relay links
  - ▷ Background: cheap power amplifiers vs. high PAPR in OFDM
- Focus:
  - ▷ Fixed infrastructure-based relay node
  - ▷ Amplify and forward (AF) protocol
  - ▷ Frequency-domain processing



System Model

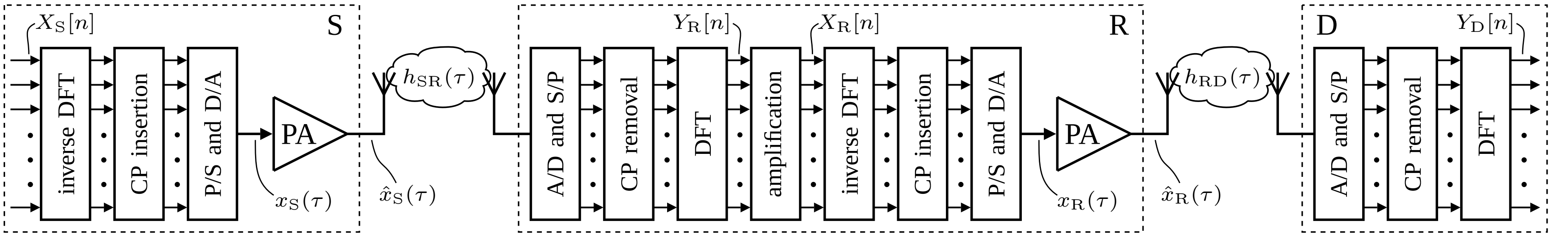
OFDM Signal with Nonlinear Distortion



Signal model from transmitter $t \in \{\mathrm{S}, \mathrm{R}\}$ to receiver $r \in \{\mathrm{R}, \mathrm{D}\}$

- Tx: $X_t[n]$ in frequency domain, $x_t(\tau)$ in time domain
- Nonlinear power amplifier (PA) — static and memoryless:
  $\hat{x}_t(\tau) = f_t\left(x_t(\tau)\right) = K_t x_t(\tau) + v_t(\tau)$, i.e., $\hat{X}_t[n] = K_t X_t[n] + V_t[n]$
  - ▷ Power of distortion noise is $\varsigma_t^2 = \frac{1}{P_t}\mathcal{E}\{|V_t[n]|^2\}$
- Multipath channel: $h_{tr}(\tau)$ and in frequency domain $H_{tr}[n]$
- Rx: $Y_r[n] = K_t H_{tr}[n] X_t[n] + H_{tr} V_t[n] + W_r[n]$

Two-Hop Amplify-and-Forward OFDM Relay Link



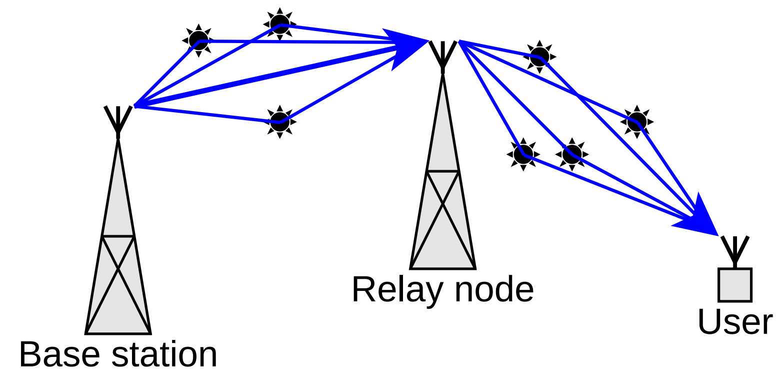
Parallel processing of subcarriers in frequency domain

- Amplification by $\beta[n] = \sqrt{\frac{P_R}{(K_S^2+\varsigma_S^2)P_S|H_{SR}[n]|^2+\sigma_R^2}}$
- End-to-end signal-to-noise ratio (SNR) becomes

$$\gamma[n] = \frac{\frac{K_S^2\gamma_{SR}[n]}{\varsigma_S^2\gamma_{SR}[n]+1} \cdot \frac{K_R^2\gamma_{RD}[n]}{\varsigma_R^2\gamma_{RD}[n]+1}}{\frac{K_S^2\gamma_{SR}[n]}{\varsigma_S^2\gamma_{SR}[n]+1} + \frac{K_R^2\gamma_{RD}[n]}{\varsigma_R^2\gamma_{RD}[n]+1} + 1} \quad \text{where} \quad \begin{cases} \gamma_{SR}[n] = \frac{P_S|H_{SR}[n]|^2}{\sigma_R^2} \\ \gamma_{RD}[n] = \frac{P_R|H_{RD}[n]|^2}{\sigma_D^2} \end{cases}$$

Multipath Channels

The source and the relay are fixed nodes, and the destination is mobile



- Source–relay channel $h_{\mathrm{SR}}(\tau)$: stationary multipath components
  - ▷ Practical model: $\gamma_{\mathrm{SR}}[n] = \bar{\gamma}_{\mathrm{SR}}[n] = \mathcal{E}\left\{\gamma_{\mathrm{SR}}[n]\right\}$
  - ▷ Not necessarily line-of-sight, i.e., $\bar{\gamma}_{\mathrm{SR}}[n] \neq \bar{\gamma}_{\mathrm{SR}}[m]$
- Relay–destination channel $h_{\mathrm{RD}}(\tau)$: Rayleigh fading components
  - ▷ SNR distribution $f_{\gamma_{\mathrm{RD}}[n]}(s) = (1/\bar{\gamma}_{\mathrm{RD}}[n]) \exp\left(-s/\bar{\gamma}_{\mathrm{RD}}[n]\right)$

BEP Analysis

BEP Derivation

- Reformulate the end-to-end SNR as $\gamma[n] = 2\frac{\Theta[n]\gamma_{\mathrm{RD}}[n]}{\Omega[n]\gamma_{\mathrm{RD}}[n]+1}$ in which

$$\Theta[n] = \frac{1}{2}\frac{K_{\mathrm{S}}^2 K_{\mathrm{R}}^2 \bar{\gamma}_{\mathrm{SR}}[n]}{\left(K_{\mathrm{S}}^2 + \varsigma_{\mathrm{S}}^2\right)\bar{\gamma}_{\mathrm{SR}}[n] + 1}$$

$$\Omega[n] = \frac{\left(K_{\mathrm{S}}^2 \varsigma_{\mathrm{R}}^2 + \varsigma_{\mathrm{S}}^2 K_{\mathrm{R}}^2 + \varsigma_{\mathrm{S}}^2 \varsigma_{\mathrm{R}}^2\right)\bar{\gamma}_{\mathrm{SR}}[n] + K_{\mathrm{R}}^2 + \varsigma_{\mathrm{R}}^2}{\left(K_{\mathrm{S}}^2 + \varsigma_{\mathrm{S}}^2\right)\bar{\gamma}_{\mathrm{SR}}[n] + 1}$$

- Omitting few steps, average bit-error probability (BEP) is calculated as

$$\bar{P}_e[n] = \frac{1}{2} - \frac{1}{2\sqrt{\pi}\Omega[n]\bar{\gamma}_{\mathrm{RD}}[n]} \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma\left(k+\frac{3}{2}\right)}{k!\left(k+\frac{1}{2}\right)} \times \left(\frac{\Theta[n]}{\Omega[n]}\right)^{k+\frac{1}{2}} U\left(k+\frac{3}{2}, 2, \frac{1}{\Omega[n]\bar{\gamma}_{\mathrm{RD}}[n]}\right)$$

BEP Bounds

- Linear PAs by substituting $K_{\mathrm{S}} = K_{\mathrm{R}} = 1$ and $\varsigma_{\mathrm{S}}^2 = \varsigma_{\mathrm{R}}^2 = 0$
- Asymptotic lower bounds
  - ▷ First hop limited by distortion: $\bar{P}_e[n] \geq \bar{P}_e^{\bar{\gamma}_{\mathrm{SR}}\to\infty}[n]$ with

$$\Theta^{\bar{\gamma}_{\mathrm{SR}}\to\infty}[n] = \lim_{\bar{\gamma}_{\mathrm{SR}}[n]\to\infty} \Theta[n] = \frac{1}{2}\frac{K_{\mathrm{S}}^2 K_{\mathrm{R}}^2}{K_{\mathrm{S}}^2 + \varsigma_{\mathrm{S}}^2},$$

$$\Omega^{\bar{\gamma}_{\mathrm{SR}}\to\infty}[n] = \lim_{\bar{\gamma}_{\mathrm{SR}}[n]\to\infty} \Omega[n] = \frac{K_{\mathrm{S}}^2\varsigma_{\mathrm{R}}^2 + \varsigma_{\mathrm{S}}^2 K_{\mathrm{R}}^2 + \varsigma_{\mathrm{S}}^2\varsigma_{\mathrm{R}}^2}{K_{\mathrm{S}}^2 + \varsigma_{\mathrm{S}}^2}$$

    Simpler bound by linear PAs: $\bar{P}_e^{\bar{\gamma}_{\mathrm{SR}}\to\infty}[n] \geq \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\bar{\gamma}_{\mathrm{RD}}[n]}{\bar{\gamma}_{\mathrm{RD}}[n]+2}}$
  - ▷ Second hop limited by distortion:

$$\bar{P}_e[n] \geq \bar{P}_e^{\bar{\gamma}_{\mathrm{RD}}\to\infty}[n] = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{\Theta[n]}{\Omega[n]}}\right) \geq \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{\bar{\gamma}_{\mathrm{SR}}[n]}{2}}\right)$$

Discussion

Soft Limiter PA Model

Transmitter t

- Clipping the amplitude of PA input signal:

$$|\hat{x}_t(\tau)| = f_t\left(|x_t(\tau)|\right) = \begin{cases} \frac{A_t}{\nu_t}|x_t(\tau)|, & |x_t(\tau)| \leq \nu_t\sqrt{P_t} \\ A_t\sqrt{P_t}, & |x_t(\tau)| > \nu_t\sqrt{P_t} \end{cases}$$

  - ▷ Adjustable input back-off ν_t^2
- PA factors in closed form:

$$K_t = \frac{A_t}{\nu_t}\left(1 - \exp\left(-\nu_t^2\right) + \frac{\sqrt{\pi}\nu_t}{2}\operatorname{erfc}\left(\nu_t\right)\right)$$

$$\varsigma_t^2 = \frac{A_t^2}{\nu_t^2}\left(1 - \exp\left(-\nu_t^2\right)\right) - K_t^2$$

Optimization of Back-off Factors (1)

$(\nu_{\mathrm{S}}^*, \nu_{\mathrm{R}}^*) = \arg\min_{(\nu_{\mathrm{S}},\nu_{\mathrm{R}})} \bar{P}_e[n]$
subject to $\nu_{\mathrm{S}} > 0$, $\nu_{\mathrm{R}} > 0$

- Can be decomposed as

$$\nu_{\mathrm{S}}^* = \arg\min_{\nu_{\mathrm{S}}} \frac{K_{\mathrm{S}}^2 \bar{\gamma}_{\mathrm{SR}}[n]}{\varsigma_{\mathrm{S}}^2 \bar{\gamma}_{\mathrm{SR}}[n] + 1}, \quad \nu_{\mathrm{S}} > 0$$

$$\nu_{\mathrm{R}}^* = \arg\min_{\nu_{\mathrm{R}}} \bar{P}_e[n]\Big|_{\nu_{\mathrm{S}}=\nu_{\mathrm{S}}^*}, \quad \nu_{\mathrm{R}} > 0$$

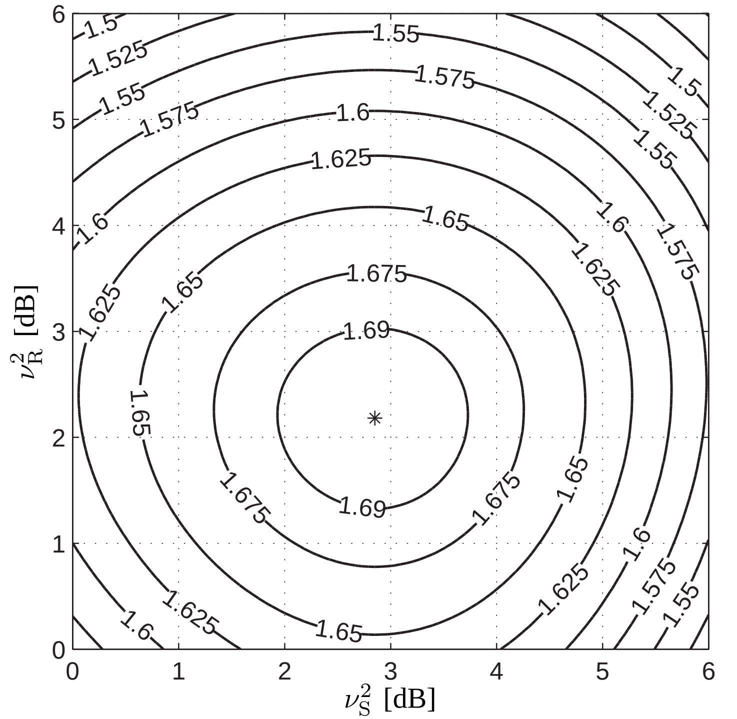
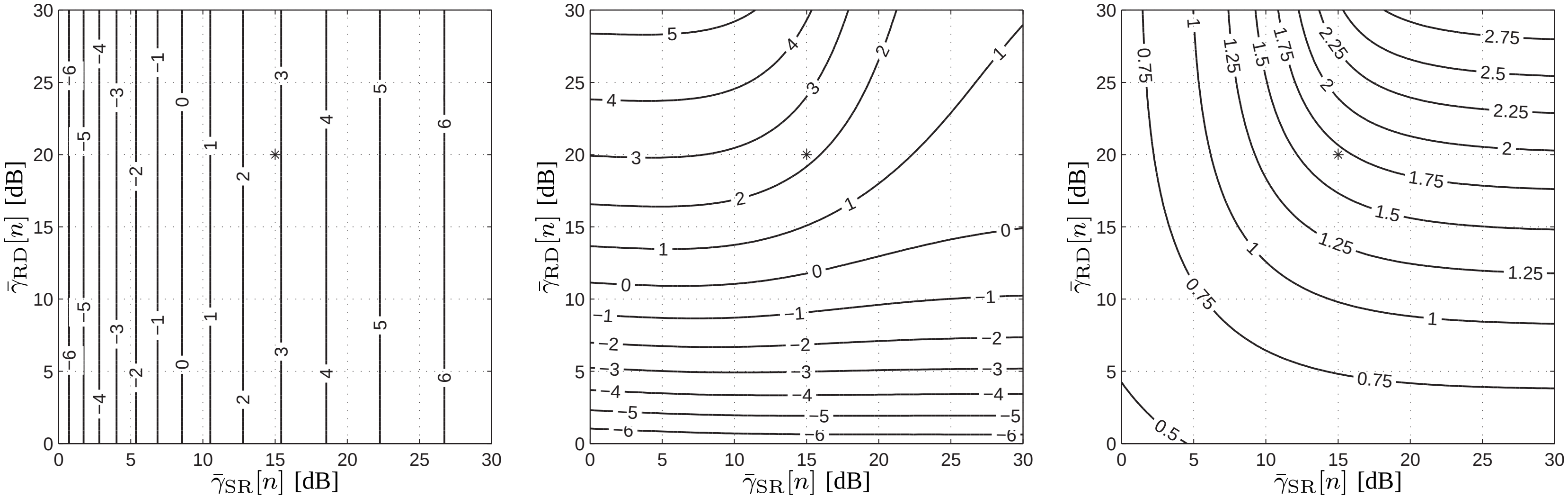


Fig. 2. Contour plot for $-\log_{10}\left(\bar{P}_e[n]\right)$ in terms of the input back-off factors when $\bar{\gamma}_{\mathrm{SR}}[n] = 15$dB, $\bar{\gamma}_{\mathrm{RD}}[n] = 20$dB, and $A_{\mathrm{S}} = A_{\mathrm{R}} = 1$. The minimal bit error probability $\bar{P}_e^*[n] = 2.0 \cdot 10^{-2}$ is reached with $\nu_{\mathrm{S}}^{*2} = 2.85$dB and $\nu_{\mathrm{R}}^{*2} = 2.18$dB (marked with $*$).

Optimization of Back-off Factors (2)



(a) ν_{S}^{*2} [dB] (b) ν_{R}^{*2} [dB] (c) $-\log_{10}\left(\bar{P}_e^*[n]\right)$

Fig. 3. Contour plots for the optimized input back-off factors and the minimal bit error probability when $A_{\mathrm{S}} = A_{\mathrm{R}} = 1$. The SNR pair ($\bar{\gamma}_{\mathrm{SR}}[n]$,$\bar{\gamma}_{\mathrm{RD}}[n]$) considered in Fig. 2 is marked with $*$.

- Numerical optimization validates the decomposition

Numerical Results (1)

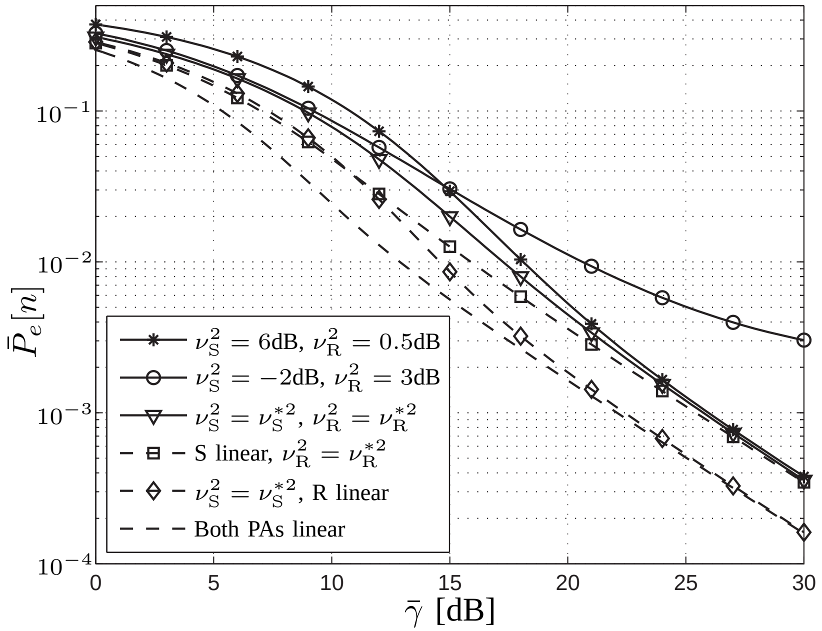


Fig. 4. Average bit error probability when $\bar{\gamma}_{SR}[n] = \bar{\gamma}$, $\bar{\gamma}_{RD}[n] = \bar{\gamma} + 5$dB and $A_S = A_R = 1$.

- Input back-off adjustment is a trade-off between having closely optimal BEP at low SNR or a low BEP floor at high SNR

Numerical Results (2)

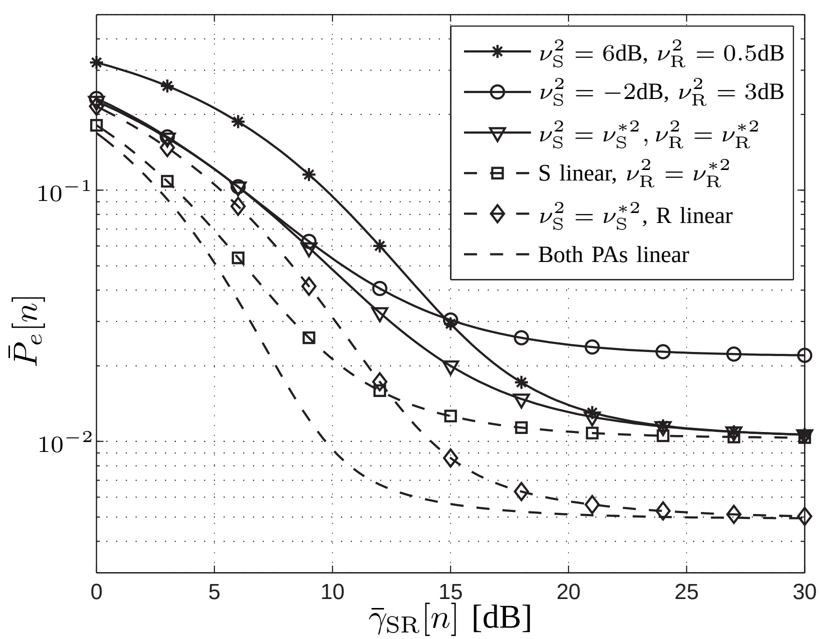


Fig. 5. Average bit error probability in terms of the first hop SNR when the second hop SNR $\bar{\gamma}_{RD}[n] = 20$dB and $A_S = A_R = 1$.

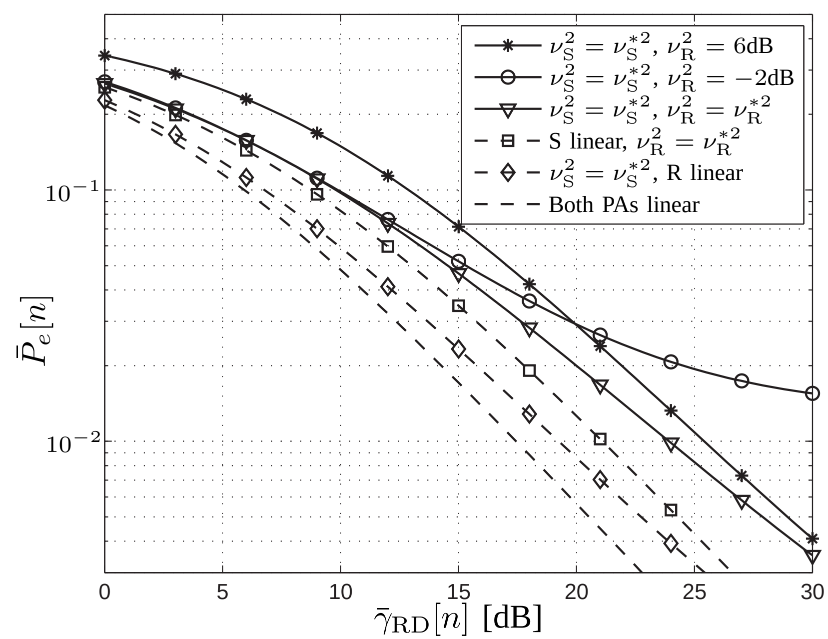


Fig. 6. Average bit error probability in terms of the second hop SNR when the first hop SNR $\bar{\gamma}_{SR}[n] = 15$dB (hence $\nu_S^{*2} = 2.85$dB) and $A_S = A_R = 1$.

- PA nonlinearity causes both SNR loss and higher BEP floors
- The performance is asymmetric: The second hop is more critical

Conclusion

Conclusion

- Derivation of closed-form bit error probability expressions
  - ▷ Infrastructure-based amplify-and-forward OFDM relay link
  - ▷ The effect of nonlinear power amplifiers
  - ▷ The performance with ideal linear PAs is a special case
- The results are applicable to any memoryless power amplifier
  - ▷ Soft limiter model was selected for the numerical illustrations
- The adjustment of power amplifier input back-offs

Thank you!

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