SINR Analysis of Full-Duplex OFDM Repeaters

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Introduction

- Full-duplex relay a.k.a. on-channel repeater a.k.a. gap filler
  - Patching coverage holes cost-efficiently
  - Boosting cell edge coverage by reducing path losses

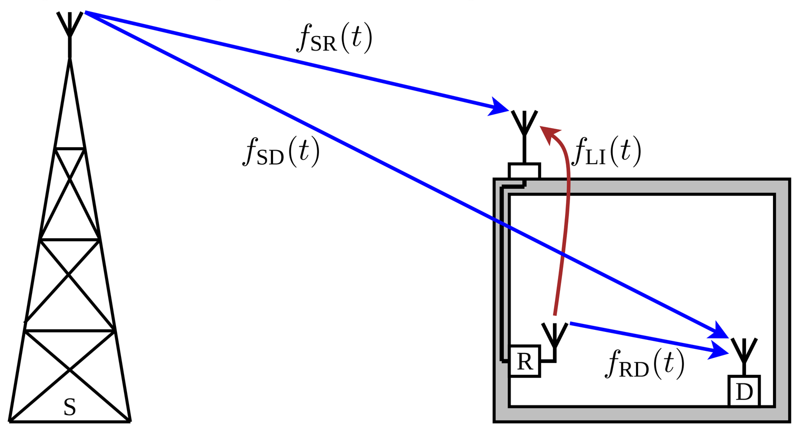


Fig. 1. Two-hop full-duplex relay link with loop interference in a typical downlink outdoor-to-indoor transmission scenario. The power–delay profiles of the channels are represented by $f_{\mathrm{SR}}(t)$, $f_{\mathrm{LI}}(t)$, $f_{\mathrm{RD}}(t)$, and $f_{\mathrm{SD}}(t)$.

- The main technical challenge is the *loop interference*, i.e., crosstalk between transmission and reception of the relay
  - In practice, spatially-separated transmit and receive antennas are required
  - Classification of countermeasures
    1) Physical isolation between the relay antennas
    2) Directivity properties of the antennas
    3) Signal processing for loop interference cancellation
    4) Relay gain optimization (NEW in this presentation!)

System model

- Standard OFDM transmission
  - Length of the fast Fourier transform T_{FFT}
  - Length of the cyclic prefix T_{CP}
  - Time of reference τ_{TOR} marks the start of symbol demodulation
- The frequency-selective multipath channels are specified in terms of *power-delay profiles (PDPs)*
  - The classic (single-)exponential PDP

$$f_1(t) = f_1(t, g, \tau, \sigma) = \frac{g}{\sigma} e^{-\frac{t-\tau}{\sigma}} \mathrm{U}(t-\tau)$$

  * gain $g = \int_{-\infty}^{\infty} f_1(t)dt$, mean delay $\mu = \frac{1}{g}\int_{-\infty}^{\infty} t f_1(t)dt = \sigma + \tau$, mean square delay spread $\sigma^2 = \frac{1}{g}\int_{-\infty}^{\infty}(t-\mu)^2 f_1(t)dt$
  - The *new double-exponential PDP* by convolution

$$f_2(t) = f_2(t, g, \tau, \sigma_1, \sigma_2) = \int_{-\infty}^{\infty} f_1(z, g_1, \tau_1, \sigma_1) f_1(t-z, g_2, \tau_2, \sigma_2)dz = \frac{\sigma_1}{\sigma_1-\sigma_2} f_1(t, g, \tau, \sigma_1) + \frac{\sigma_2}{\sigma_2-\sigma_1} f_1(t, g, \tau, \sigma_2)$$

  * combined delay $\tau = \tau_1 + \tau_2$, total end-to-end gain $g = \int_{-\infty}^{\infty} f_2(t)dt = g_1 g_2$
- The full-duplex relay with gain G_{R}
  - The PDP of the channel from transmitter $\mathfrak{t}$ to receiver $\mathfrak{r}$

$$f_{\mathfrak{tr}}(t) = \mathcal{E}\{|h_{\mathfrak{tr}}(t)|^2\} = \sum_{i=1}^{N_{\mathfrak{tr}}} f_1\left(t, g_{\mathfrak{tr}}[i], \tau_{\mathfrak{tr}}[i], \sigma_{\mathfrak{tr}}[i]\right)$$

  * $N_{\mathfrak{tr}}$ clusters: gain, delay and RMS delay spread of the ith cluster $g_{\mathfrak{tr}}[i]$, $\tau_{\mathfrak{tr}}[i]$ and $\sigma_{\mathfrak{tr}}[i]$
  * Total channel gain $G_{\mathfrak{tr}} = \int_{-\infty}^{\infty} f_{\mathfrak{tr}}(t)dt = \sum_{i=1}^{N_{\mathfrak{tr}}} g_{\mathfrak{tr}}[i]$
  - The PDP of the loop interference channel $f_{\mathrm{LI}}(t) = G_{\mathrm{LI}}\delta(t-\tau_{\mathrm{LI}})$
  * Single impulse with gain G_{LI} and delay τ_{LI}, because the main source of loop interference is the direct coupling between the directive transmit and receive antennas
  - The end-to-end PDP becomes

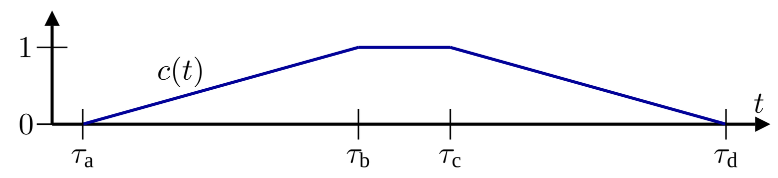
$$f(t) = \sum_{i=1}^{N_{\mathrm{SD}}} f_1\left(t, g_{\mathrm{SD}}[i], \tau_{\mathrm{SD}}[i], \sigma_{\mathrm{SD}}[i]\right) + \sum_{n=0}^{\infty}\sum_{i=1}^{N_{\mathrm{SR}}}\sum_{j=1}^{N_{\mathrm{RD}}} f_2(t, g_{\mathrm{tot}}[n,i,j], \tau_{\mathrm{tot}}[n,i,j], \sigma_{\mathrm{SR}}[i], \sigma_{\mathrm{RD}}[j])$$

  * $g_{\mathrm{tot}}[n,i,j] = g_{\mathrm{SR}}[i] G_{\mathrm{R}} (G_{\mathrm{LI}} G_{\mathrm{R}})^n g_{\mathrm{RD}}[j]$, $\tau_{\mathrm{tot}}[n,i,j] = \tau_{\mathrm{SR}}[i] + \tau_{\mathrm{R}} + (\tau_{\mathrm{LI}} + \tau_{\mathrm{R}})n + \tau_{\mathrm{RD}}[j]$
  * The relay gain G_{R} must be limited by $G_{\mathrm{R}} < \frac{1}{G_{\mathrm{LI}}}$
  - Relay receive signal power $P_{\mathrm{S}} G_{\mathrm{SR}} + P_{\mathrm{R}} G_{\mathrm{LI}} + \rho_{\mathrm{R}}^2$
  * Transmit power in the source P_{S}
  - Relay transmit signal power $P_{\mathrm{R}} = \frac{(P_{\mathrm{S}} G_{\mathrm{SR}} + \rho_{\mathrm{R}}^2) G_{\mathrm{R}}}{1 - G_{\mathrm{LI}} G_{\mathrm{R}}}$
  - Total signal power in the destination $P_{\mathrm{tot}} = P_{\mathrm{S}} G_{\mathrm{SD}} + P_{\mathrm{S}} \frac{G_{\mathrm{SR}} G_{\mathrm{R}} G_{\mathrm{RD}}}{1 - G_{\mathrm{LI}} G_{\mathrm{R}}}$
  - Total noise power in the destination $P_{\mathrm{N}} = \frac{G_{\mathrm{R}} G_{\mathrm{RD}}}{1 - G_{\mathrm{LI}} G_{\mathrm{R}}} \rho_{\mathrm{R}}^2 + \rho_{\mathrm{D}}^2$
  * Assuming relay receiver noise power ρ_{R}^2 and destination receiver noise power ρ_{D}^2

End-to-end SINR

- Signal-to-interference and noise ratio (SINR) is $\gamma = \frac{P_{\mathrm{U}}}{P_{\mathrm{I}} + P_{\mathrm{N}}}$
  - Useful signal power $P_{\mathrm{U}} = P_{\mathrm{S}} \int_{-\infty}^{\infty} c^2(t) f(t) dt$
  - Interference power $P_{\mathrm{I}} = P_{\mathrm{tot}} - P_{\mathrm{U}}$
  - Weighting function for OFDM:

$$c(t) = \max\left\{0, \min\left\{1, \frac{t-\tau_{\mathrm{a}}}{T_{\mathrm{FFT}}}, \frac{\tau_{\mathrm{d}}-t}{T_{\mathrm{FFT}}}\right\}\right\}$$



  * $\tau_{\mathrm{a}} = \tau_{\mathrm{TOR}} - T_{\mathrm{FFT}}$, $\tau_{\mathrm{b}} = \tau_{\mathrm{TOR}}$, $\tau_{\mathrm{c}} = \tau_{\mathrm{TOR}} + T_{\mathrm{CP}}$, and $\tau_{\mathrm{d}} = \tau_{\mathrm{TOR}} + T_{\mathrm{CP}} + T_{\mathrm{FFT}}$
- Useful signal power in the relay link
  - By defining $\mathcal{I}(g, \tau, \sigma) = \int_{-\infty}^{\infty} c^2(t) f_1(t, g, \tau, \sigma) dt$:

$$\frac{P_{\mathrm{U}}}{P_{\mathrm{S}}} = \sum_{i=1}^{N_{\mathrm{SD}}} \mathcal{I}\left(g_{\mathrm{SD}}[i], \tau_{\mathrm{SD}}[i], \sigma_{\mathrm{SD}}[i]\right) + \sum_{n=0}^{\infty}\sum_{i=1}^{N_{\mathrm{SR}}}\sum_{j=1}^{N_{\mathrm{RD}}}\left(\frac{\mathcal{I}(g_{\mathrm{tot}}[n,i,j], \tau_{\mathrm{tot}}[n,i,j], \sigma_{\mathrm{SR}}[i])}{1 - \sigma_{\mathrm{RD}}[j]/\sigma_{\mathrm{SR}}[i]} + \frac{\mathcal{I}(g_{\mathrm{tot}}[n,i,j], \tau_{\mathrm{tot}}[n,i,j], \sigma_{\mathrm{RD}}[j])}{1 - \sigma_{\mathrm{SR}}[i]/\sigma_{\mathrm{RD}}[j]}\right)$$

  - After integrations omitted here:

$$\mathcal{I}(g, \tau, \sigma) = \mathcal{I}_{\mathrm{ab}}(g, \tau, \sigma)\mathrm{U}(\tau_{\mathrm{b}} - \tau) + \mathcal{I}_{\mathrm{bc}}(g, \tau, \sigma)\mathrm{U}(\tau_{\mathrm{c}} - \tau) + \mathcal{I}_{\mathrm{cd}}(g, \tau, \sigma)\mathrm{U}(\tau_{\mathrm{d}} - \tau)$$

$$\mathcal{I}_{\mathrm{ab}}(g, \tau, \sigma) = \frac{g e^{\frac{\tau}{\sigma}}\left(\left(\sigma^2 + (\max\{\tau, \tau_{\mathrm{a}}\} - \tau_{\mathrm{a}} + \sigma)^2\right) e^{-\frac{\max\{\tau, \tau_{\mathrm{a}}\}}{\sigma}} - \left(\sigma^2 + (\tau_{\mathrm{b}} - \tau_{\mathrm{a}} + \sigma)^2\right) e^{-\frac{\tau_{\mathrm{b}}}{\sigma}}\right)}{T_{\mathrm{FFT}}^2}$$

$$\mathcal{I}_{\mathrm{bc}}(g, \tau, \sigma) = g e^{\frac{\tau}{\sigma}}\left(e^{-\frac{\max\{\tau, \tau_{\mathrm{b}}\}}{\sigma}} - e^{-\frac{\tau_{\mathrm{c}}}{\sigma}}\right)$$

$$\mathcal{I}_{\mathrm{cd}}(g, \tau, \sigma) = \frac{g e^{\frac{\tau}{\sigma}}\left(\left(\sigma^2 + (\max\{\tau, \tau_{\mathrm{c}}\} - \tau_{\mathrm{d}} + \sigma)^2\right) e^{-\frac{\max\{\tau, \tau_{\mathrm{c}}\}}{\sigma}} - 2\sigma^2 e^{-\frac{\tau_{\mathrm{d}}}{\sigma}}\right)}{T_{\mathrm{FFT}}^2}$$

- Signal powers and SINR in an example setup:

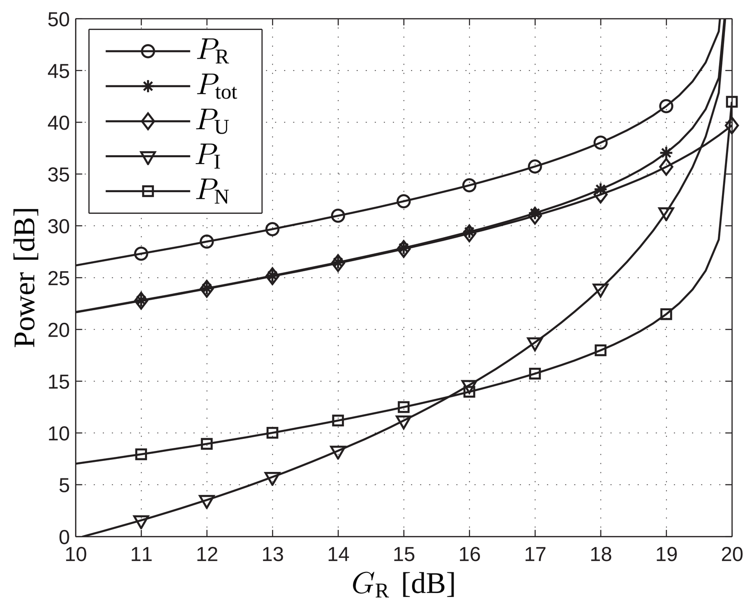


Fig. 3. The powers of the signal components in the full-duplex relay link in terms of the relay gain when $G_{\mathrm{LI}} = -20$dB. The length of the cyclic prefix is selected as $T_{\mathrm{CP}} = 12\mu$s

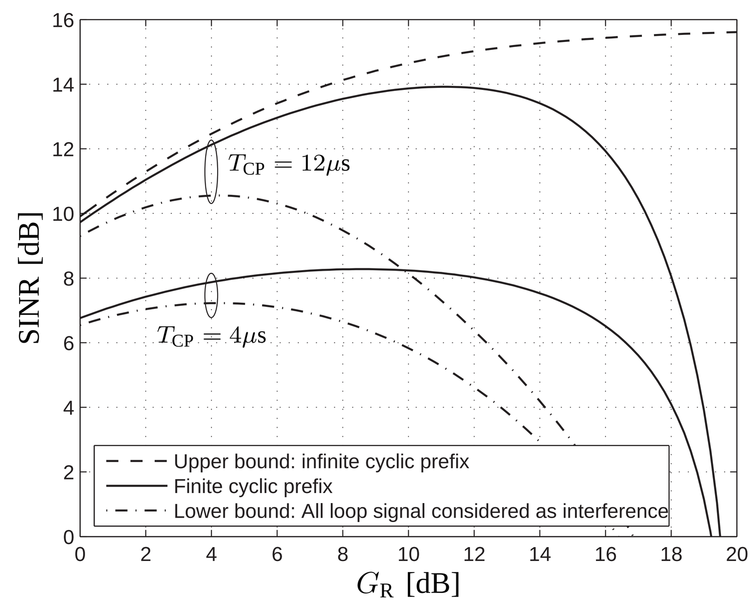


Fig. 4. Signal-to-interference and noise ratio in the full-duplex relay link in terms of the relay gain when $G_{\mathrm{LI}} = -20$dB.

Relay gain optimization

- With large relay gain, the end-to-end PDP decays slowly due to the feedback loop, and *more multipath components are transferred outside the cyclic prefix* causing interference
  → Interference power may increase faster than the useful signal power
  - Optimal gain

$$G_{\mathrm{R}}^{\mathrm{OPT}} = \arg\max_{G_{\mathrm{R}}} \gamma$$

  - Gain margin approach

$$G_{\mathrm{R}}^{\mathrm{GM}} = \frac{1}{\Delta_{\mathrm{GM}} G_{\mathrm{LI}}}$$

  * Margin Δ_{GM} usually 10–15dB
  - Power normalization approach

$$G_{\mathrm{R}}^{\mathrm{PWR}} = \frac{P_{\mathrm{R}}}{P_{\mathrm{S}} G_{\mathrm{SR}} + P_{\mathrm{R}} G_{\mathrm{LI}} + \rho_{\mathrm{R}}^2}$$

  * Pre-selected transmit power P_{R}

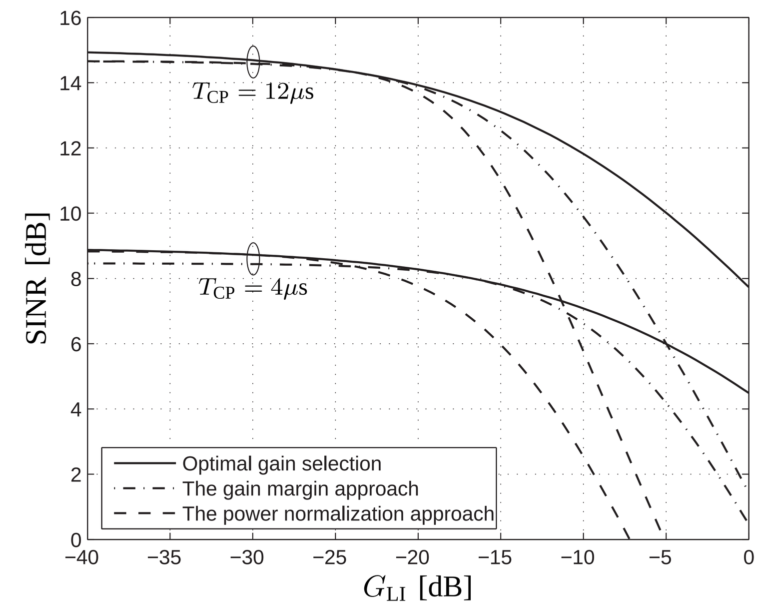


Fig. 5. Signal-to-interference and noise ratio in the full-duplex relay link in terms of the residual loop interference channel gain with the different gain selection methods. We choose $\Delta_{\mathrm{GM}} = 10$dB and $P_{\mathrm{R}} = 30$dB for the gain margin and power normalization approaches, respectively.

⇒ Optimization of the relay gain guarantees proper transmit power usage and minimizes the effect of the loop interference

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