Outage Probabilities in Infrastructure-Based Single-Frequency Relay Links

Taneli Riihonen, Stefan Werner, Risto Wichman, and Jyri Hämäläinen
Helsinki University of Technology, Finland

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Outline

Introduction

- Relays offer a cost-effective solution for network coverage extension and gap filling in cellular networks
  - Amplify-and-forward (AF): variable gain (VG) or fixed gain (FG)
  - Decode-and-forward (DF)
- A single-cell setup, where a base station (BS) communicates with a mobile user equipment (UE) via an infrastructure relay node (RN)
  - Spectral efficiency can be improved over conventional half-duplex operation by applying frequency reuse $\Longrightarrow$ single-frequency relaying
    - The pre-log factor $1/2$ in capacity is avoided
    - Signal quality degradation due to the interfering first-hop transmission and imperfect loop interference cancellation
- Outage probability expressions facilitate performance analysis that includes the effects of loop interference and frequency reuse

Single-frequency relay link

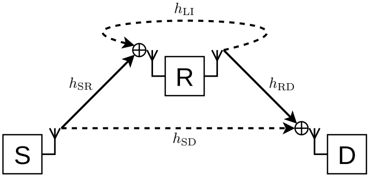

Fig. 1. System model of a two-hop single-frequency relay link.

- The signal model:

$$r[i] = h_{\rm SR}x[i] + h_{\rm LI}t[i] + n_{\rm R}[i]$$
$$t[i] = \begin{cases} \beta r[i-\tau], & \text{with AF} \\ \tilde{x}[i-\tau], & \text{with DF} \end{cases}$$
$$y[i] = h_{\rm RD}t[i] + h_{\rm SD}x[i] + n_{\rm D}[i]$$

  - The instantaneous channel SNRs are defined as $\gamma_{\rm SR} = |h_{\rm SR}|^2/\sigma_{\rm R}^2$, $\gamma_{\rm RD} = |h_{\rm RD}|^2/\sigma_{\rm D}^2$, $\gamma_{\rm SD} = |h_{\rm SD}|^2/\sigma_{\rm D}^2$, and $\gamma_{\rm LI} = |h_{\rm LI}|^2/\sigma_{\rm R}^2$
  - The average channel SNRs are defined as $\bar{\gamma}_{\rm SR} = \mathcal{E}_h\{|h_{\rm SR}|^2\}/\sigma_{\rm R}^2$, $\bar{\gamma}_{\rm RD} = \mathcal{E}_h\{|h_{\rm RD}|^2\}/\sigma_{\rm D}^2$, $\bar{\gamma}_{\rm SD} = \mathcal{E}_h\{|h_{\rm SD}|^2\}/\sigma_{\rm D}^2$, and $\bar{\gamma}_{\rm LI} = \mathcal{E}_h\{|h_{\rm LI}|^2\}/\sigma_{\rm R}^2$

End-to-end SINR in amplify-and-forward

- Variable gain (VG) and fixed gain (FG)

$$\beta = \begin{cases} \left(|h_{\mathrm{SR}}|^2 + |h_{\mathrm{LI}}|^2 + \sigma_{\mathrm{R}}^2\right)^{-\frac{1}{2}}, & \text{with VG} \\ \left(\mathcal{E}_h\{|h_{\mathrm{SR}}|^2\} + |h_{\mathrm{LI}}|^2 + \sigma_{\mathrm{R}}^2\right)^{-\frac{1}{2}}, & \text{with FG} \end{cases}$$

- The destination receive power:

$$\mathcal{E}_x\{|y[i]|^2\} = |h_{\mathrm{SR}}|^2\beta^2|h_{\mathrm{RD}}|^2 + \left(|h_{\mathrm{SR}}|^2 + \sigma_{\mathrm{R}}^2\right)\beta^2|h_{\mathrm{RD}}|^2 \frac{|h_{\mathrm{LI}}|^2\beta^2}{1 - |h_{\mathrm{LI}}|^2\beta^2} + |h_{\mathrm{SD}}|^2 + \beta^2|h_{\mathrm{RD}}|^2\sigma_{\mathrm{R}}^2 + \sigma_{\mathrm{D}}^2$$

- The end-to-end SINR:

$$\gamma = \frac{|h_{\mathrm{SR}}|^2|h_{\mathrm{RD}}|^2}{\left(\frac{(|h_{\mathrm{SR}}|^2+\sigma_{\mathrm{R}}^2)|h_{\mathrm{LI}}|^2}{1/\beta^2 - |h_{\mathrm{LI}}|^2} + \sigma_{\mathrm{R}}^2\right)|h_{\mathrm{RD}}|^2 + \frac{|h_{\mathrm{SD}}|^2+\sigma_{\mathrm{D}}^2}{\beta^2}} = \begin{cases} \frac{\gamma_{\mathrm{R}}\gamma_{\mathrm{D}}}{\gamma_{\mathrm{R}}+\gamma_{\mathrm{D}}+1}, & \text{with VG} \\ \frac{\gamma_{\mathrm{R}}\gamma_{\mathrm{D}}}{\bar{\gamma}_{\mathrm{R}}+\gamma_{\mathrm{D}}+1}, & \text{with FG} \end{cases}$$

where $\gamma_{\mathrm{R}} = \frac{\gamma_{\mathrm{SR}}}{\gamma_{\mathrm{LI}}+1}$, $\bar{\gamma}_{\mathrm{R}} = \frac{\bar{\gamma}_{\mathrm{SR}}}{\gamma_{\mathrm{LI}}+1}$ and $\gamma_{\mathrm{D}} = \frac{\gamma_{\mathrm{RD}}}{\gamma_{\mathrm{SD}}+1}$

Outage probability

- The probability that the end-to-end relay link cannot support a desired performance level defined by a threshold SINR $\gamma_{\text{th}} > 0$

$$P_{\text{out}}(\gamma_{\text{th}}) = \begin{cases} P(\gamma < \gamma_{\text{th}}) = F_\gamma(\gamma_{\text{th}}), & \text{with AF} \\ P(\min\{\gamma_{\text{R}}, \gamma_{\text{D}}\} < \gamma_{\text{th}}) & \text{with DF} \end{cases}$$

  - With AF, the outage probability is given by the cumulative distribution function (CDF), F_γ, of the end-to-end SINR
  - With DF, an outage is happens, if either of the hops is in outage, i.e., the performance is always determined by the weaker hop
- Infrastructure-based relay link
  - BS–RN, RN–BS, and loop interference channels are static
  - RN–UE, UE–RN, BS–UE, UE–BS channels are Rayleigh fading

DL: Amplify-and-forward

- In downlink, the first hop is static and the second hop is fading, i.e.,

$$
\begin{aligned}
F_{\gamma_{\mathrm{R}}}(s) &= \begin{cases} 0, & s < \bar{\gamma}_{\mathrm{R}} \\ 1, & s \geq \bar{\gamma}_{\mathrm{R}} \end{cases} \\
F_{\gamma_{\mathrm{D}}}(s) &= \int_0^{\infty} F_{\gamma_{\mathrm{RD}}}(s\,u + s)\, f_{\gamma_{\mathrm{SD}}}(u) du = 1 - \frac{\bar{\gamma}_{\mathrm{RD}}}{\bar{\gamma}_{\mathrm{RD}} + \bar{\gamma}_{\mathrm{SD}} s} e^{-\frac{s}{\bar{\gamma}_{\mathrm{RD}}}}
\end{aligned}
$$

- VG and FG are equal in DL
- The outage probability becomes

$$
\begin{aligned}
P_{\mathrm{out}}(\gamma_{\mathrm{th}}) &= \begin{cases} 1, & \gamma_{\mathrm{th}} > \bar{\gamma}_{\mathrm{R}} \\ F_{\gamma_{\mathrm{D}}}\left(\frac{[\bar{\gamma}_{\mathrm{R}}+1]\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathrm{R}}-\gamma_{\mathrm{th}}}\right), & \text{otherwise} \end{cases} \\
&= 1 - \frac{1}{1 + \frac{(\bar{\gamma}_{\mathrm{R}}+1)\gamma_{\mathrm{th}}}{(\bar{\gamma}_{\mathrm{R}}-\gamma_{\mathrm{th}})\bar{\gamma}_{\mathrm{RD}}}\bar{\gamma}_{\mathrm{SD}}} e^{-\frac{(\bar{\gamma}_{\mathrm{R}}+1)\gamma_{\mathrm{th}}}{(\bar{\gamma}_{\mathrm{R}}-\gamma_{\mathrm{th}})\bar{\gamma}_{\mathrm{RD}}}}
\end{aligned}
$$

for $\gamma_{\mathrm{th}} < \bar{\gamma}_{\mathrm{R}}$, and $P_{\mathrm{out}}(\gamma_{\mathrm{th}}) = 1$ for $\gamma_{\mathrm{th}} \geq \bar{\gamma}_{\mathrm{R}}$

DL: Decode-and-forward

- For decode-and-forward

$$P_{\mathrm{out}}(\gamma_{\mathrm{th}}) = P(\min\{\gamma_{\mathrm{R}}, \gamma_{\mathrm{D}}\} < \gamma_{\mathrm{th}})$$

- Outage probability becomes

$$\begin{aligned} P_{\mathrm{out}}(\gamma_{\mathrm{th}}) &= 1 - [1 - F_{\gamma_{\mathrm{R}}}(\gamma_{\mathrm{th}})][1 - F_{\gamma_{\mathrm{D}}}(\gamma_{\mathrm{th}})] \\ &= \begin{cases} 1, & \gamma_{\mathrm{th}} > \bar{\gamma}_{\mathrm{R}} \\ 1 - \frac{1}{1 + \frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathrm{RD}}}\bar{\gamma}_{\mathrm{SD}}} e^{-\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathrm{RD}}}}, & \text{otherwise} \end{cases} \end{aligned}$$

UL: Amplify-and-forward

- In uplink, the first hop is fading and the second hop is static, i.e.,

$$F_{\gamma_{\mathrm{R}}}(s) = 1 - e^{-\frac{\bar{\gamma}_{\mathrm{LI}}+1}{\bar{\gamma}_{\mathrm{SR}}}s} = 1 - e^{-\frac{s}{\bar{\gamma}_{\mathrm{R}}}}$$

$$F_{\gamma_{\mathrm{D}}}(s) = \begin{cases} 1, & s > \bar{\gamma}_{\mathrm{RD}} \\ 1 - F_{\gamma_{\mathrm{SD}}}\left(\frac{\bar{\gamma}_{\mathrm{RD}}-s}{s}\right), & \text{otherwise} \end{cases} = \begin{cases} 1, & s > \bar{\gamma}_{\mathrm{RD}} \\ e^{-\frac{\bar{\gamma}_{\mathrm{RD}}-s}{\bar{\gamma}_{\mathrm{SD}}s}}, & \text{otherwise} \end{cases}$$

- The outage probability
  - Variable gain:

$$P_{\mathrm{out}}(\gamma_{\mathrm{th}}) = 1 - \int_0^{\frac{\bar{\gamma}_{\mathrm{RD}}-\gamma_{\mathrm{th}}}{\gamma_{\mathrm{th}}}} \left\{1 - F_{\gamma_{\mathrm{R}}}\left[\gamma_{\mathrm{R}}(u,\gamma_{\mathrm{th}})\right]\right\} f_{\gamma_{\mathrm{SD}}}(u)du$$

  - Fixed gain:

$$P_{\mathrm{out}}(\gamma_{\mathrm{th}}) = 1 - \frac{1}{1 + \frac{(\bar{\gamma}_{\mathrm{R}}+1)\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathrm{R}}\bar{\gamma}_{\mathrm{RD}}}\bar{\gamma}_{\mathrm{SD}}} e^{-\frac{\bar{\gamma}_{\mathrm{R}}+\bar{\gamma}_{\mathrm{RD}}+1}{\bar{\gamma}_{\mathrm{R}}\bar{\gamma}_{\mathrm{RD}}}\gamma_{\mathrm{th}}}$$

UL: Decode-and-forward

- As before, for decode-and-forward

$$P_{\mathrm{out}}(\gamma_{\mathrm{th}}) = P(\min\{\gamma_{\mathrm{R}}, \gamma_{\mathrm{D}}\} < \gamma_{\mathrm{th}})$$

- The outage probability is simply

$$\begin{aligned} P_{\mathrm{out}}(\gamma_{\mathrm{th}}) &= 1 - [1 - F_{\gamma_{\mathrm{R}}}(\gamma_{\mathrm{th}})]\,[1 - F_{\gamma_{\mathrm{D}}}(\gamma_{\mathrm{th}})] \\ &= P_{\mathrm{out}}(\gamma_{\mathrm{th}}) = 1 - (1 - e^{-\frac{\bar{\gamma}_{\mathrm{RD}} - \gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathrm{SD}}\gamma_{\mathrm{th}}}})e^{-\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathrm{R}}}} \end{aligned}$$

for $\gamma_{\mathrm{th}} \leq \bar{\gamma}_{\mathrm{RD}}$ and $P_{\mathrm{out}}(\gamma_{\mathrm{th}}) = 1$ for $\gamma_{\mathrm{th}} > \bar{\gamma}_{\mathrm{RD}}$

Comparison of the schemes

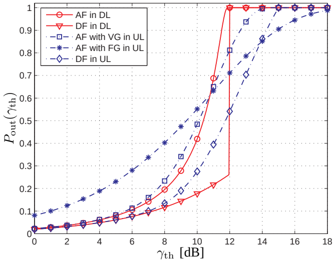

Fig. 2. Outage probability at the mid-SNR range. The average BS–RN/RN–BS channel SNRs are 15dB, the average RN–UE/UE–RN channel SNRs are 20dB, and $\bar{\gamma}_{\text{SD}} = \bar{\gamma}_{\text{LI}} = 0$dB. The markers illustrate the simulated values.

- The behavior in DL and in UL is quite different
  - In DL, the end-to-end SINR is highly limited by the first hop SINR
  - The limitation due to the RN–BS channel SNR is less critical in UL
  - FG protocol is worse than VG protocol in UL (equivalent in DL)
  - DF protocol is better than AF protocol

The effect of loop interference

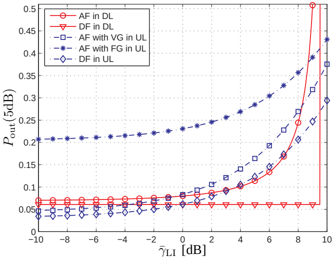

Fig. 3. Outage probability for threshold SINR $\gamma_{\text{th}} = 5\text{dB}$ with varying loop interference power. The average BS–RN/RN–BS channel SNRs are 15dB, the average RN–UE/UE–RN channel SNRs are 20dB, and $\bar{\gamma}_{\text{SD}} = 0\text{dB}$. The markers illustrate the simulated values.

- The FG protocol is clearly inferior to the other protocols in UL
- Loop interference limitation is more strict in DL than in UL
- The effect of the interfering signal is small, if the interference power does not exceed the receiver noise power with more than 3dB

The effect of interference due to frequency reuse

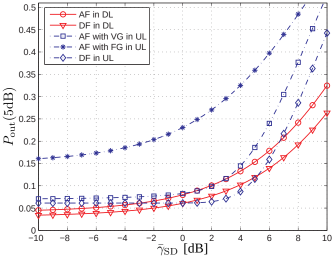

Fig. 4. Outage probability for threshold SINR $\gamma_{\mathrm{th}} = 5\mathrm{dB}$ with varying interference power due to frequency reuse. The average BS–RN/RN–BS channel SNRs are 15dB, the average RN–UE/UE–RN channel SNRs are 20dB, and $\bar{\gamma}_{\mathrm{LI}} = 0\mathrm{dB}$. The markers illustrate the simulated values.

- The high-power interference due to frequency reuse has more deteriorating effect on UL than on DL
- The effect of the interfering signal is small, if the interference power does not exceed the receiver noise power with more than 3dB

Conclusion

- Performance analysis of an infrastructure-based relay link used for cell coverage extension
  - Amplify-and-forward with fixed or variable gain
  - Decode-and-forward
  - Both downlink and uplink
- An attractive alternative is to let the relay operate in a single-frequency mode
  - The increase in spectral efficiency comes at the expense of additional interference due to frequency reuse and nonideal isolation between relay transmitter and receiver
- Derivation of new closed-form expressions for the outage probability
  - Comparison of the different relaying protocols
  - The single-frequency mode can be implemented with only minor increase in outage probability, if the interfering signals are not considerably stronger than the receiver noise

Thank you!

- Questions?
- Discussion?