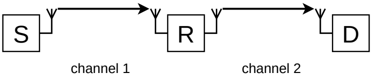
Comparison of Full-Duplex and Half-Duplex Modes with a Fixed Amplify-and-Forward Relay

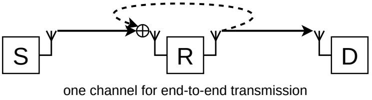
Taneli Riihonen, Stefan Werner, and Risto Wichman
Helsinki University of Technology, Finland

IEEE WCNC, Budapest, Hungary
April 06, 2009

Outline

Introduction to relaying modes





- Half-Duplex (HD)
  - *Pre-log 1/2 in capacity*
  - Mobile relays and cooperative communication
    - Also with a single antenna

- Full-Duplex (FD)
  - *Loop interference*
  - Fixed infrastructure-based relays
    - Separate rx and tx antennas
    - Loop cancellation algorithms

How to select the optimal mode?

Amplify-and-forward full-duplex relay link

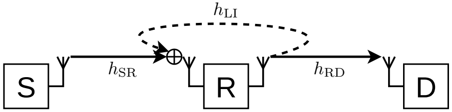


Fig. 1. Two-hop relay link with potential loop interference.

- The signal model:

$$
\begin{aligned}
r[i] &= h_{\rm SR} x[i] + h_{\rm LI} t[i] + n_{\rm R}[i] \\
t[i] &= \beta r[i-\tau] = \beta \sum_{j=1}^{\infty} (h_{\rm LI}\beta)^{j-1} \left(h_{\rm SR} x[i-j\tau] + n_{\rm R}[i-j\tau]\right) \\
y[i] &= h_{\rm RD} t[i] + n_{\rm D}[i]
\end{aligned}
$$

- Amplification by $\beta = \left(|h_{\rm SR}|^2 + |h_{\rm LI}|^2 + \sigma_{\rm R}^2\right)^{-1/2}$ due to transmit power normalization $\mathcal{E}_x\{|t[i]|^2\} = 1$, where

$$
\mathcal{E}_x\{|t[i]|^2\} = \beta^2 \sum_{j=1}^{\infty} \left(|h_{\rm LI}|^2 \beta^2\right)^{j-1} \left(|h_{\rm SR}|^2 + \sigma_{\rm R}^2\right) = \beta^2 \frac{|h_{\rm SR}|^2 + \sigma_{\rm R}^2}{1 - |h_{\rm LI}|^2 \beta^2}
$$

End-to-end SINR

- Parametrization in terms of channel SNRs:
  - Instantaneous: $\gamma_{\mathrm{SR}} = |h_{\mathrm{SR}}|^2/\sigma_{\mathrm{R}}^2$, $\gamma_{\mathrm{RD}} = |h_{\mathrm{RD}}|^2/\sigma_{\mathrm{D}}^2$, $\gamma_{\mathrm{LI}} = |h_{\mathrm{LI}}|^2/\sigma_{\mathrm{R}}^2$
  - Mean: $\bar{\gamma}_{\mathrm{SR}} = \mathcal{E}_h\{|h_{\mathrm{SR}}|^2\}/\sigma_{\mathrm{R}}^2$, $\bar{\gamma}_{\mathrm{RD}} = \mathcal{E}_h\{|h_{\mathrm{RD}}|^2\}/\sigma_{\mathrm{D}}^2$, $\bar{\gamma}_{\mathrm{LI}} = \mathcal{E}_h\{|h_{\mathrm{LI}}|^2\}/\sigma_{\mathrm{R}}^2$
- The received power in the destination

$$\mathcal{E}_x\{|y[i]|^2\} = \underbrace{|h_{\mathrm{SR}}|^2\beta^2|h_{\mathrm{RD}}|^2}_{\text{useful signal}} + \underbrace{\left(|h_{\mathrm{SR}}|^2 + \sigma_{\mathrm{R}}^2\right)\beta^2|h_{\mathrm{RD}}|^2\frac{|h_{\mathrm{LI}}|^2\beta^2}{1-|h_{\mathrm{LI}}|^2\beta^2}}_{\text{loop interference}} + \underbrace{\beta^2|h_{\mathrm{RD}}|^2\sigma_{\mathrm{R}}^2 + \sigma_{\mathrm{D}}^2}_{\text{noise}}$$

- The instantaneous end-to-end SINR:

$$\gamma = \frac{|h_{\mathrm{SR}}|^2|h_{\mathrm{RD}}|^2}{\left(\frac{(|h_{\mathrm{SR}}|^2+\sigma_{\mathrm{R}}^2)|h_{\mathrm{LI}}|^2}{1/\beta^2-|h_{\mathrm{LI}}|^2} + \sigma_{\mathrm{R}}^2\right)|h_{\mathrm{RD}}|^2 + \frac{\sigma_{\mathrm{D}}^2}{\beta^2}} = \frac{\bar{\gamma}_{\mathrm{SR}}\gamma_{\mathrm{RD}}}{\bar{\gamma}_{\mathrm{SR}} + (\gamma_{\mathrm{RD}}+1)(\bar{\gamma}_{\mathrm{LI}}+1)}$$

Average capacities (1)

- Infrastructure-based relay link
  - The source and the relay are fixed and the destination is mobile
  - SR and LI channels are modeled as static (AWGN)
  - RD channel is modeled with Rayleigh fading
- Calculation of the average capacities ($\bar{\mathcal{C}} = \mathcal{E}_h\{\log_2(1+\gamma)\}$)
  - In the full-duplex mode:

$$\bar{\mathcal{C}}_{\mathrm{FD}} = \frac{e^{\frac{1}{\bar{\gamma}_{\mathrm{RD}}}} E_1\left(\frac{1}{\bar{\gamma}_{\mathrm{RD}}}\right) - e^{\frac{\bar{\gamma}_{\mathrm{SR}}+\bar{\gamma}_{\mathrm{LI}}+1}{\bar{\gamma}_{\mathrm{RD}}(\bar{\gamma}_{\mathrm{LI}}+1)}} E_1\left(\frac{\bar{\gamma}_{\mathrm{SR}}+\bar{\gamma}_{\mathrm{LI}}+1}{\bar{\gamma}_{\mathrm{RD}}(\bar{\gamma}_{\mathrm{LI}}+1)}\right)}{\log_e(2)}$$

  - In the half-duplex mode ($\bar{\gamma}_{\mathrm{LI}} = 0$ and pre-log factor 1/2):

$$\bar{\mathcal{C}}_{\mathrm{HD}} = \frac{e^{\frac{1}{\bar{\gamma}_{\mathrm{RD}}}} E_1\left(\frac{1}{\bar{\gamma}_{\mathrm{RD}}}\right) - e^{\frac{\bar{\gamma}_{\mathrm{SR}}+1}{\bar{\gamma}_{\mathrm{RD}}}} E_1\left(\frac{\bar{\gamma}_{\mathrm{SR}}+1}{\bar{\gamma}_{\mathrm{RD}}}\right)}{2\log_e(2)}$$

Average capacities (2)

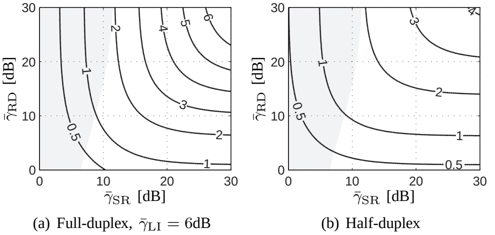


Fig. 2. Contour plots for the average capacity [bit/s/Hz] with the transmission modes. The shaded region illustrates the SNR area where capacity with the half-duplex mode is higher than that with the full-duplex mode.

- There is a clear trade-off between the modes
  - How large is the capacity gain due to proper mode selection?
  - In which SNR region one mode if preferred over the other?

Mode selection

- Analytical rules for selecting the best mode
  - Proposition 1: If $\bar{\gamma}_{\mathrm{SR}} > \bar{\gamma}_{\mathrm{LI}}^2 - 1$, then $\bar{\mathcal{C}}_{\mathrm{FD}} > \bar{\mathcal{C}}_{\mathrm{HD}}$ for all $\bar{\gamma}_{\mathrm{RD}}$
    - Full-duplex is the best at high SNR
  - Corollary 1: If $\bar{\gamma}_{\mathrm{LI}} < 0\mathrm{dB}$, then $\bar{\mathcal{C}}_{\mathrm{FD}} > \bar{\mathcal{C}}_{\mathrm{HD}}$ for all $\bar{\gamma}_{\mathrm{SR}}$ and $\bar{\gamma}_{\mathrm{RD}}$
    - The loop interference may be embedded in the receiver noise
  - Proposition 2: If $\bar{\gamma}_{\mathrm{SR}} < \bar{\gamma}_{\mathrm{LI}} - 1$, then $\bar{\mathcal{C}}_{\mathrm{FD}} < \bar{\mathcal{C}}_{\mathrm{HD}}$ for all $\bar{\gamma}_{\mathrm{RD}}$
    - Half-duplex is the best at low SNR
  - When $\bar{\gamma}_{\mathrm{LI}} - 1 \leq \bar{\gamma}_{\mathrm{SR}} \leq \bar{\gamma}_{\mathrm{LI}}^2 - 1$, the choice depends on $\bar{\gamma}_{\mathrm{RD}}$
    - The mid-SNR range

Capacity ratio of the modes

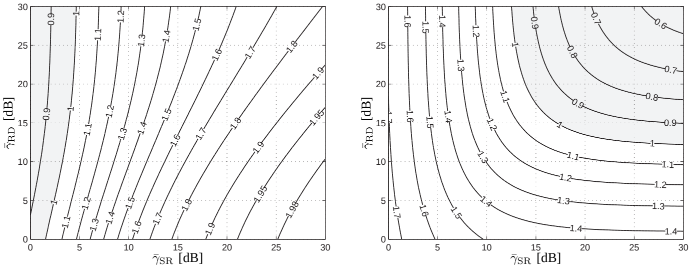


Fig. 3. Contour plot for the capacity ratio $\bar{\mathcal{C}}_{\mathrm{FD}}/\bar{\mathcal{C}}_{\mathrm{HD}}$ when $\bar{\gamma}_{\mathrm{LI}} = 3$dB. The shaded region illustrates the SNR area where capacity with the half-duplex mode is higher than that with the full-duplex mode.

Fig. 5. Contour plot for the capacity ratio $\bar{\mathcal{C}}_{\mathrm{FD}}/\bar{\mathcal{C}}_{\mathrm{HD}}$ when $\bar{\gamma}_{\mathrm{SR}}/\bar{\gamma}_{\mathrm{LI}} = 6$dB. The shaded region illustrates the SNR area where capacity with the half-duplex mode is higher than that with the full-duplex mode.

- When $\bar{\gamma}_{\mathrm{LI}} = 3$dB:
  - $\bar{\mathcal{C}}_{\mathrm{FD}} < \bar{\mathcal{C}}_{\mathrm{HD}}$, if $\bar{\gamma}_{\mathrm{SR}} < 0$dB (Proposition 2)
  - $\bar{\mathcal{C}}_{\mathrm{FD}} > \bar{\mathcal{C}}_{\mathrm{HD}}$, if $\bar{\gamma}_{\mathrm{SR}} > 4.7$dB (Proposition 1)

Break-even loop interference power (1)

- The capacity trade-off
  - $\bar{\mathcal{C}}_{\mathrm{FD}} \to 2\bar{\mathcal{C}}_{\mathrm{HD}}$ when $\bar{\gamma}_{\mathrm{LI}} \to 0$
  - $\bar{\mathcal{C}}_{\mathrm{FD}} \to 0$ when $\bar{\gamma}_{\mathrm{LI}} \to \infty$
  - For which $\bar{\gamma}_{\mathrm{LI}} = \bar{\Gamma}_{\mathrm{LI}}$ the capacities are equal ($\bar{\mathcal{C}}_{\mathrm{FD}} = \bar{\mathcal{C}}_{\mathrm{HD}}$)?
    - Numerically solving $\bar{\gamma}_{\mathrm{LI}}$ from

$$\frac{1}{2}\left[E_1\left(\frac{1}{\bar{\gamma}_{\mathrm{RD}}}\right) + e^{\frac{\bar{\gamma}_{\mathrm{SR}}}{\bar{\gamma}_{\mathrm{RD}}}} E_1\left(\frac{\bar{\gamma}_{\mathrm{SR}}+1}{\bar{\gamma}_{\mathrm{RD}}}\right)\right] = e^{\frac{\bar{\gamma}_{\mathrm{SR}}/\bar{\gamma}_{\mathrm{RD}}}{\bar{\gamma}_{\mathrm{LI}}+1}} E_1\left(\frac{\bar{\gamma}_{\mathrm{SR}}+\bar{\gamma}_{\mathrm{LI}}+1}{\bar{\gamma}_{\mathrm{RD}}(\bar{\gamma}_{\mathrm{LI}}+1)}\right)$$

  - $\bar{\mathcal{C}}_{\mathrm{FD}} > \bar{\mathcal{C}}_{\mathrm{HD}}$ when $\bar{\gamma}_{\mathrm{LI}} < \bar{\Gamma}_{\mathrm{LI}}$

Break-even loop interference power (2)

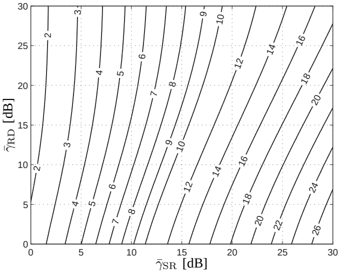


Fig. 4. Contour plot for the maximum $\bar{\gamma}_{\mathrm{LI}}$ [dB] for which $\bar{\mathcal{C}}_{\mathrm{FD}} \geq \bar{\mathcal{C}}_{\mathrm{HD}}$.

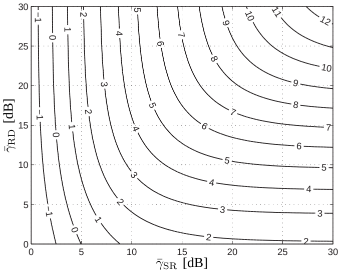


Fig. 6. Contour plot for the minimum $\bar{\gamma}_{\mathrm{SR}}/\bar{\gamma}_{\mathrm{LI}}$ [dB] for which $\bar{\mathcal{C}}_{\mathrm{FD}} \geq \bar{\mathcal{C}}_{\mathrm{HD}}$.

- $\bar{\mathcal{C}}_{\mathrm{FD}} > \bar{\mathcal{C}}_{\mathrm{HD}}$ when $\bar{\gamma}_{\mathrm{LI}} < \bar{\Gamma}_{\mathrm{LI}}$
- The power of the loop interference in FD can be quite high, because the effect of having pre-log factor 1/2 in HD is severe

SNR gain of the FD mode

- The minimum SNRs needed to achieve capacity of 1bit/s/Hz:

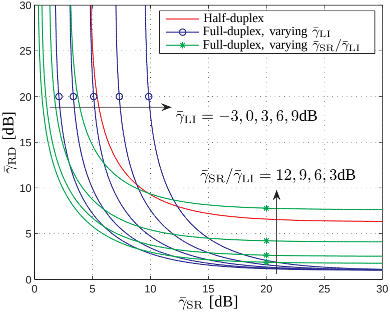


Fig. 7. Contour plot illustrating the SNR value pairs ($\bar{\gamma}_{\mathrm{SR}}$,$\bar{\gamma}_{\mathrm{RD}}$) that result in average capacity $\bar{C} = 1$bit/s/Hz.

- Depending on the loop interference power, the FD mode achieves the same capacity as the HD mode with up to 5dB smaller SNRs

Numerical examples

- In a system where $\bar{\gamma}_{\mathrm{SR}}$ and $\bar{\gamma}_{\mathrm{RD}}$ are 10–15dB:
  - Capacity with the HD mode is 1.1–1.7bit/s/Hz
  - Capacity with the FD mode is 1.1–2.3bit/s/Hz if $\bar{\gamma}_{\mathrm{LI}} = 6$dB
  - When $\bar{\gamma}_{\mathrm{LI}} = 3$dB or $\bar{\gamma}_{\mathrm{SR}}/\bar{\gamma}_{\mathrm{LI}} = 6$dB, the FD mode achieves 32%–67% or 5%–30% higher capacity than the HD mode
  - The FD mode has approximately the same capacity as the HD mode
    - if the loop interference power is as high as 6.1–10.7dB relatively to the relay input noise power
    - if the desired signal power in the relay input is as low as 3.2–5.5dB relatively to the loop interference power
- The full-duplex mode is preferable in this example system, if the loop interference power can be suppressed below the calculated limits

Conclusion

- The choice between full-duplex and half-duplex relaying modes represents a fundamental capacity trade-off
- The loop interference can be suppressed to a tolerable level in fixed infrastructure-based relays
  - It may be better to allow some SINR degradation with the full-duplex mode than to allocate two channels for eliminating the loop interference with the half-duplex mode
- Derivation of closed-form end-to-end capacities
  - Evaluation of the capacity improvement due to the full-duplex mode
  - Solving for the SNR ranges in which one mode outperforms the other
- The full-duplex mode can be superior with practical SNR values

Thank you!

- Questions?
- Discussion?