Co-phasing Full-Duplex Relay Link with Non-Ideal Feedback Information

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Introduction

- Relays for extending coverage areas or enhancing hotspot capacity
  - The amplify-and-forward protocol
  - A fixed infrastructure-based relay
- New co-phasing scheme for full-duplex relays
- Performance analysis
  - The performance improvement due to co-phasing
    - Modelling a non-ideal feedback channel
  - The effect of power control
  - Full-duplex mode vs. half-duplex mode
    - Maximum ratio combining gain vs. coherent combining gain

System model of the full-duplex relay link

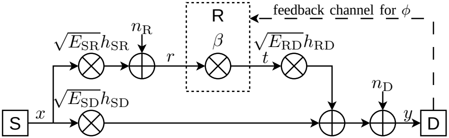


Fig. 1. System model of the full-duplex co-phasing relay link.

- A superposition of direct and relayed signals at the destination
  - Nakagami-m fading channels
- Amplify-and-forward relay with phase rotation:

$$\beta = e^{j\phi}\sqrt{\frac{P_{\rm R}}{E_{\rm SR}P_{\rm S}+\sigma_{\rm R}^2}}$$

  - A feedback channel for the phase information

Instantaneous end-to-end SNR

- The instantaneous end-to-end SNR can be formulated as

$$\gamma^{\mathrm{FD}} = \underbrace{\frac{\gamma_{\mathrm{SR}}\gamma_{\mathrm{RD}} + (\bar{\gamma}_{\mathrm{SR}} + 1)\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SR}} + \gamma_{\mathrm{RD}} + 1}}_{=\gamma^{\mathrm{incoh}}} + \underbrace{\frac{2\sqrt{\bar{\gamma}_{\mathrm{SR}} + 1}\sqrt{\gamma_{\mathrm{SR}}}\sqrt{\gamma_{\mathrm{RD}}}\sqrt{\gamma_{\mathrm{SD}}}\cos(\psi)}{\bar{\gamma}_{\mathrm{SR}} + \gamma_{\mathrm{RD}} + 1}}_{=\gamma^{\mathrm{coh}}}$$

  - γ_{xy} variables represent the channels SNRs
- The first term is the SNR with orthogonal direct and relay channels
- The second term is the SNR change due to constructive or destructive combining of the signals
  - depends on the phase difference $\psi = \angle(h_{\mathrm{SR}} h_{\mathrm{RD}}) - \angle(h_{\mathrm{SD}}) + \phi$
- Co-phasing: constructive combining by controlling ϕ

Co-phasing algorithm

- A non-ideal feedback channel for the co-phasing factor
  - N_{ph} feedback bits: quantization with $2^{N_{\mathrm{ph}}}$ uniform levels
  - Feedback bit error probability δ
- The destination selects the feedback codeword

$$\phi^* = \arg\max_{\phi\in\Phi} \gamma^{\mathrm{FD}} = \arg\max_{\phi\in\Phi} |\angle(h_{\mathrm{SR}} h_{\mathrm{RD}}) - \angle(h_{\mathrm{SD}}) + \phi|$$

  where $\Phi = \{\frac{2\pi n}{2^{N_{\mathrm{ph}}}} \mid n = 0, 1, \ldots, 2^{N_{\mathrm{ph}}} - 1\}$.
- With an error-free feedback channel, the phase difference ψ becomes uniformly distributed in $[-\frac{\pi}{2^{N_{\mathrm{ph}}}}, \frac{\pi}{2^{N_{\mathrm{ph}}}}]$

Power allocation algorithm

- In general, performance is limited by the weaker of the two hops
  - SNR balancing by using more transmit power at the weaker hop
- Determining the power allocation factor p
  - $P_{\mathrm{R}} = pP_{\mathrm{S}}$
  - Normalization of the total transmit power: $P_{\mathrm{S}} + P_{\mathrm{R}} = 1$
  - Thus, $P_{\mathrm{S}} = \frac{1}{p+1}$ and $P_{\mathrm{R}} = \frac{p}{p+1}$
- The optimal power allocation:

$$p^* = \arg\max_{p>0} \bar{\gamma}^{\mathrm{FD}}$$

  - Now the maximum is solved numerically
  - Nearly optimal closed-form solutions are available in literature

Average end-to-end SNR

- The average end-to-end SNR can be calculated as

$$\bar{\gamma}^{\mathrm{FD}} = \mathcal{E}\left[\gamma^{\mathrm{FD}}\right] = \bar{\gamma}^{\mathrm{incoh}} + \bar{\gamma}^{\mathrm{coh}}$$

- Average SNR with incoherent relaying:

$$\bar{\gamma}^{\mathrm{incoh}} = \bar{\gamma}_{\mathrm{SD}} + (\bar{\gamma}_{\mathrm{SR}} - \bar{\gamma}_{\mathrm{SD}}) m_{\mathrm{RD}} e^{\frac{\bar{\gamma}_{\mathrm{SR}}+1}{\bar{\gamma}_{\mathrm{RD}}/m_{\mathrm{RD}}}} E_{m_{\mathrm{RD}}+1}\left(\frac{\bar{\gamma}_{\mathrm{SR}}+1}{\bar{\gamma}_{\mathrm{RD}}/m_{\mathrm{RD}}}\right)$$

- The SNR improvement due to co-phasing:

$$\bar{\gamma}^{\mathrm{coh}} = 2\,\frac{B_{\mathrm{SR}} B_{\mathrm{RD}} B_{\mathrm{SD}}}{\sqrt{\bar{\gamma}_{\mathrm{RD}}/m_{\mathrm{RD}}}} \sqrt{\frac{\bar{\gamma}_{\mathrm{SR}}+1}{\bar{\gamma}_{\mathrm{RD}}/m_{\mathrm{RD}}}}\, e^{\frac{\bar{\gamma}_{\mathrm{SR}}+1}{\bar{\gamma}_{\mathrm{RD}}/m_{\mathrm{RD}}}} E_{m_{\mathrm{RD}}+\frac{1}{2}}\left(\frac{\bar{\gamma}_{\mathrm{SR}}+1}{\bar{\gamma}_{\mathrm{RD}}/m_{\mathrm{RD}}}\right) c_{N_{\mathrm{ph}},\delta}$$

where $B_{xy} = \sqrt{\frac{\bar{\gamma}_{xy}}{m_{xy}}}\frac{\Gamma(m_{xy}+\frac{1}{2})}{\Gamma(m_{xy})}$ and $c_{N_{\mathrm{ph}},\delta} = \mathcal{E}\left[\cos(\psi)\right]$

  - For error-free feedback channel $c_{N_{\mathrm{ph}},0} = \frac{2^{N_{\mathrm{ph}}}}{\pi}\sin(\frac{\pi}{2^{N_{\mathrm{ph}}}})$

The effect of feedback bit errors

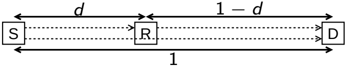
- Feedback bit error probability δ
- Standard binary reflected Gray code
  - With 1, 2 and 3 feedback bits:

$$
\begin{aligned}
c_{1,\delta} &= \frac{2}{\pi}(1-2\delta) \\
c_{2,\delta} &= \frac{2\sqrt{2}}{\pi}(1-2\delta) \\
c_{3,\delta} &= \frac{4\sqrt{2-\sqrt{2}}}{\pi}\left(1-(1-\frac{\sqrt{2}}{2})\delta\right)(1-2\delta)
\end{aligned}
$$

  - Co-phasing with 3 bit feedback already offers almost the same SNR improvement as optimal unquantized co-phasing

Example system setup

- System geometry:



  - Rayleigh channels ($m = 1$)
  - $E_{\mathrm{SD}} = 20$ dB, normalized noise power $\sigma_{\mathrm{R}}^2 = \sigma_{\mathrm{D}}^2 = 1$
  - Path loss model with exponent 3
    - $\bar{\gamma}_{\mathrm{SD}} = P_{\mathrm{S}} E_{\mathrm{SD}}$, $\bar{\gamma}_{\mathrm{SR}} = \frac{P_{\mathrm{S}} E_{\mathrm{SD}}}{d^3}$, and $\bar{\gamma}_{\mathrm{RD}} = \frac{P_{\mathrm{R}} E_{\mathrm{SD}}}{(1-d)^3}$
- Comparison to reference systems
  - Direct transmission
  - Half-duplex relaying with maximum ratio combining (MRC) of the two transmission at the destination

Power allocation

- When the relay is close to the destination, use larger transmit power in the source node
- When the relay is close to the source, use larger transmit power in the relay node
    - Except that in full-duplex relaying with co-phasing, similar transmit power in both nodes is optimal

- The optimal power allocation:

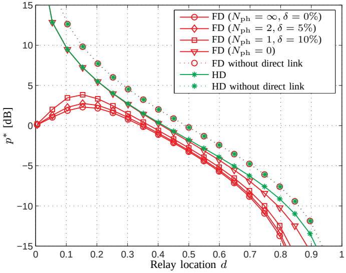


Fig. 3. The optimal power allocation factor p^* as a function of the relay location. The transmit powers are then P_S, and $P_R = p^* P_S$ for the source and the relay, respectively. The optimal power allocation for the direct link without a relay is $p^* = -\infty$ dB (not shown here).

Evaluation of average end-to-end SNR

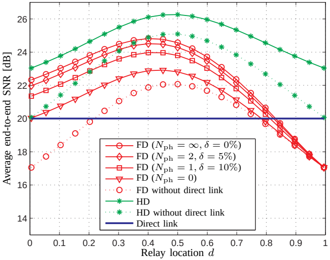


Fig. 2. The average end-to-end SNR for varying relay location with uniform power allocation ($p = 1$).

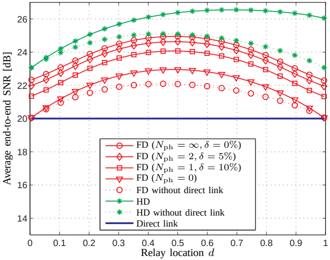


Fig. 4. The average end-to-end SNR for varying relay location with optimal power allocation ($p = p^*$).

- The coherent combining gain in full-duplex relaying is close to the MRC gain in half-duplex relaying
- When the relay is close to the destination, power allocation is crucial
- The average SNR is smaller in full-duplex than in half-duplex, but this comparison does not take into account lower end-to-end rate due to two time slots of half-duplex relaying

Evaluation of 1% outage capacity

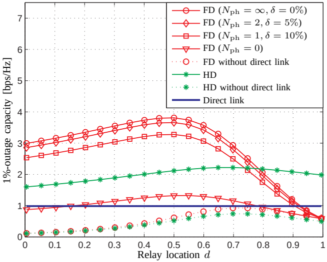


Fig. 5. The 1%-outage capacity for varying relay location with uniform power allocation ($p = 1$).

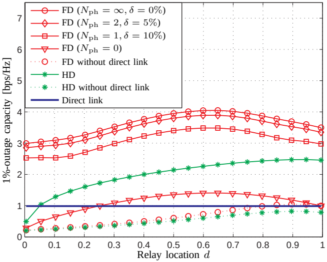


Fig. 6. The 1%-outage capacity for varying relay location with power allocation that maximizes the average end-to-end SNR ($p = p^*$).

- Already single bit co-phasing improves the performance even if the feedback bit error probability is high
- The power allocation is beneficial for making full-duplex perform well in all relay locations
- Outage capacity in full-duplex is higher than that in half-duplex

Conclusion

- The co-phasing algorithm facilitates coherent combining gain that is comparable to the MRC gain of half-duplex relaying
  - Closed-form expressions for the average end-to-end SNR
- The SNR with maximum ratio combining in half-duplex relaying is higher than the SNR with co-phasing in full-duplex relaying
- However, full-duplex relaying can achieve better capacity than half-duplex even with coarsely quantized phase information and with a large feedback bit error rate, because the rate pre-log factor $\frac{1}{2}$ of half-duplex operation is avoided

Thank you!

- Questions?
- Discussion?