Co-phasing Full-Duplex Relay Link with Non-Ideal Feedback Information

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Introduction

- Relays for extending coverage areas or enhancing hotspot capacity
  - The amplify-and-forward protocol
  - A fixed infrastructure-based relay
- New co-phasing scheme for full-duplex relays
- Performance analysis
  - The performance improvement due to co-phasing
    - Modelling a non-ideal feedback channel
  - The effect of power control
  - Full-duplex mode vs. half-duplex mode
    - Maximum ratio combining gain vs. coherent combining gain
System model of the full-duplex relay link

Fig. 1. System model of the full-duplex co-phasing relay link.

- A superposition of direct and relayed signals at the destination
  - Nakagami-\(m\) fading channels
- Amplify-and-forward relay with phase rotation:
  \[
  \beta = e^{j\phi} \sqrt{\frac{P_R}{E_{SR} P_S + \sigma_R^2}}
  \]
  - A feedback channel for the phase information
The instantaneous end-to-end SNR can be formulated as

\[ \gamma_{FD} = \frac{\gamma_{SR} \gamma_{RD} + (\bar{\gamma}_{SR} + 1)\gamma_{SD}}{\bar{\gamma}_{SR} + \gamma_{RD} + 1} + \frac{2\sqrt{\gamma_{SR} + 1} \sqrt{\gamma_{SR} \gamma_{RD} \sqrt{\gamma_{SD}}} \cos(\psi)}{\bar{\gamma}_{SR} + \gamma_{RD} + 1} \]

\[ = \bar{\gamma}_{incoh} \]

\[ = \gamma_{coh} \]

- \( \gamma_{xy} \) variables represent the channels SNRs
- The first term is the SNR with orthogonal direct and relay channels
- The second term is the SNR change due to constructive or destructive combining of the signals
  - depends on the phase difference \( \psi = \angle(h_{SR} h_{RD}) - \angle(h_{SD}) + \phi \)
- Co-phasing: constructive combining by controlling \( \phi \)
A non-ideal feedback channel for the co-phasing factor

- $N_{ph}$ feedback bits: quantization with $2^{N_{ph}}$ uniform levels
- Feedback bit error probability $\delta$

The destination selects the feedback codeword

$$\phi^* = \arg \max_{\phi \in \Phi} \gamma^{FD} = \arg \max_{\phi \in \Phi} |\angle(h_{SR} h_{RD}) - \angle(h_{SD}) + \phi|$$

where $\Phi = \{\frac{2\pi n}{2^{N_{ph}}} | n = 0, 1, \ldots, 2^{N_{ph}} - 1\}$.

With an error-free feedback channel, the phase difference $\psi$ becomes uniformly distributed in $[-\frac{\pi}{2^{N_{ph}}}, \frac{\pi}{2^{N_{ph}}}]$.
Power allocation algorithm

- In general, performance is limited by the weaker of the two hops
  - SNR balancing by using more transmit power at the weaker hop
- Determining the power allocation factor $p$
  - $P_R = pP_S$
  - Normalization of the total transmit power: $P_S + P_R = 1$
  - Thus, $P_S = \frac{1}{p+1}$ and $P_R = \frac{p}{p+1}$
- The optimal power allocation:
  \[ p^* = \arg \max_{p>0} \gamma^{FD} \]
- Now the maximum is solved numerically
- Nearly optimal closed-form solutions are available in literature
Average end-to-end SNR

- The average end-to-end SNR can be calculated as

\[ \tilde{\gamma}^{FD} = \mathcal{E} \left[ \gamma^{FD} \right] = \tilde{\gamma}^{\text{incoh}} + \tilde{\gamma}^{\text{coh}} \]

- Average SNR with incoherent relaying:

\[ \tilde{\gamma}^{\text{incoh}} = \tilde{\gamma}_{SD} + (\tilde{\gamma}_{SR} - \tilde{\gamma}_{SD}) m_{RD} e^{\frac{\tilde{\gamma}_{SR} + 1}{\tilde{\gamma}_{RD}/m_{RD}}} E_{m_{RD}+1} \left( \frac{\tilde{\gamma}_{SR} + 1}{\tilde{\gamma}_{RD}/m_{RD}} \right) \]

- The SNR improvement due to co-phasing:

\[ \tilde{\gamma}^{\text{coh}} = 2 \frac{B_{SR} B_{RD} B_{SD}}{\sqrt{\tilde{\gamma}_{RD}/m_{RD}}} \sqrt{\frac{\tilde{\gamma}_{SR} + 1}{\tilde{\gamma}_{RD}/m_{RD}}} e^{\frac{\tilde{\gamma}_{SR} + 1}{\tilde{\gamma}_{RD}/m_{RD}}} E_{m_{RD}+1} \left( \frac{\tilde{\gamma}_{SR} + 1}{\tilde{\gamma}_{RD}/m_{RD}} \right) c_{Nph,\delta} \]

where \( B_{xy} = \sqrt{\frac{\gamma_{xy}}{m_{xy}}} \Gamma(m_{xy}+\frac{1}{2}) \) and \( c_{Nph,\delta} = \mathcal{E} \left[ \cos(\psi) \right] \)

- For error-free feedback channel \( c_{Nph,0} = \frac{2^{N_{ph}}}{\pi} \sin\left(\frac{\pi}{2^{N_{ph}}}\right) \)
The effect of feedback bit errors

- Feedback bit error probability $\delta$
- Standard binary reflected Gray code
  - With 1, 2 and 3 feedback bits:
    
    \[
    c_{1,\delta} = \frac{2}{\pi} (1 - 2\delta)
    \]
    
    \[
    c_{2,\delta} = \frac{2\sqrt{2}}{\pi} (1 - 2\delta)
    \]
    
    \[
    c_{3,\delta} = \frac{4\sqrt{2 - \sqrt{2}}}{\pi} \left( 1 - (1 - \sqrt{2} \delta) \right) (1 - 2\delta)
    \]

- Co-phasing with 3 bit feedback already offers almost the same SNR improvement as optimal unquantized co-phasing
Example system setup

- System geometry:

- Rayleigh channels \((m = 1)\)
- \(E_{SD} = 20\) dB, normalized noise power \(\sigma_R^2 = \sigma_D^2 = 1\)
- Path loss model with exponent 3
  - \(\tilde{\gamma}_{SD} = P_S E_{SD}, \quad \tilde{\gamma}_{SR} = \frac{P_S E_{SD}}{d^3}, \quad \text{and} \quad \tilde{\gamma}_{RD} = \frac{P_R E_{SD}}{(1-d)^3}\)

- Comparison to reference systems
  - Direct transmission
  - Half-duplex relaying with maximum ratio combining (MRC) of the two transmission at the destination
Power allocation

- When the relay is close to the destination, use larger transmit power in the source node.
- When the relay is close to the source, use larger transmit power in the relay node.
- Except that in full-duplex relaying with co-phasing, similar transmit power in both nodes is optimal.

The optimal power allocation:

![Fig. 3. The optimal power allocation factor $p^*$ as a function of the relay location. The transmit powers are then $P_S$, and $P_R = p^* P_S$ for the source and the relay, respectively. The optimal power allocation for the direct link without a relay is $p^* = -\infty$ dB (not shown here).]
Evaluation of average end-to-end SNR

- The coherent combining gain in full-duplex relaying is close to the MRC gain in half-duplex relaying
- When the relay is close to the destination, power allocation is crucial
- The average SNR is smaller in full-duplex than in half-duplex, but this comparison does not take into account lower end-to-end rate due to two time slots of half-duplex relaying
Evaluation of 1% outage capacity

- Already single bit co-phasing improves the performance even if the feedback bit error probability is high
- The power allocation is beneficial for making full-duplex perform well in all relay locations
- Outage capacity in full-duplex is higher than that in half-duplex
The co-phasing algorithm facilitates coherent combining gain that is comparable to the MRC gain of half-duplex relaying

- Closed-form expressions for the average end-to-end SNR

The SNR with maximum ratio combining in half-duplex relaying is higher than the SNR with co-phasing in full-duplex relaying

However, full-duplex relaying can achieve better capacity than half-duplex even with coarsely quantized phase information and with a large feedback bit error rate, because the rate pre-log factor $\frac{1}{2}$ of half-duplex operation is avoided
Thank you!

- Questions?
- Discussion?