Power Allocation for a Single-Frequency Fixed-Gain Relay Network

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Introduction

- A relay receives, processes, and then retransmits radio signals in a wireless network
  - In this paper: Amplify-and-Forward (AF) relaying
  - Other: Decode-and-Forward (DF), Estimate-and-Forward (EF)
- Amplification factor for the AF relay
  - In this paper: Fixed Gain (FG) protocol
  - Other: Variable Gain (VG) protocol would adapt to the instantaneous fluctuations of the channels
- We study the simplest possible two-hop relaying network that consists of three nodes: one source, one relay and one destination node
  - The source and the relay transmit simultaneously on the same frequency band, i.e., only single frequency is allocated
- We present solutions for transmit power allocation between the source and the relay node under aggregate power constraint, when the average end-to-end signal to noise ratio (SNR) is used as a performance metric

System Model

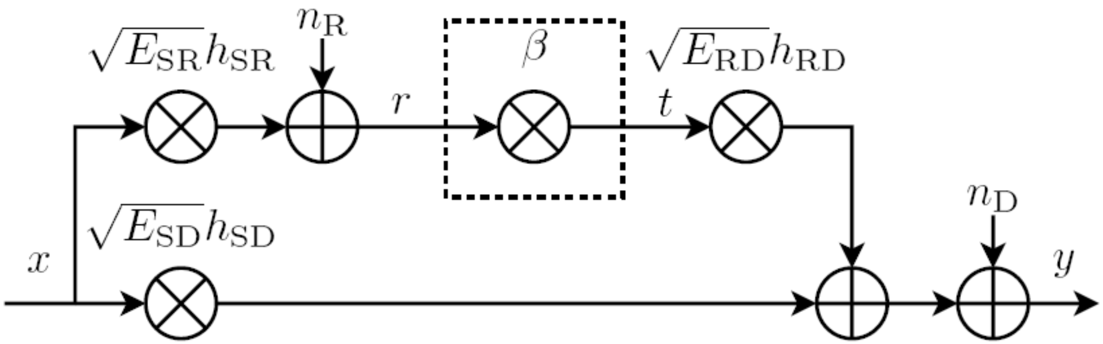


Figure 1: Single-frequency amplify-and-forward relay link.

- Fixed Gain (FG) amplify-and-forward (AF) operation:

$$\beta = \sqrt{\frac{P_{\mathrm{R}}}{E_{\mathrm{SR}}P_{\mathrm{S}} + \sigma_{\mathrm{R}}^2}}$$

- The destination receives an incoherent superposition of direct link and relayed signals
- We are not restricted to set any specific distributions for h_{SR} and h_{SD}. However, the channel h_{RD} is assumed to be Nakagami-m_{RD} distributed

Restrictions vs. Benefits of Single-Frequency Operation

- Two quite strong, but still justifiable assumptions:
  - Concurrent transmission and reception at the same frequency band in the relay
    - Appropriate avoidance of loopback interference
    - Not suitable for user cooperation, only fixed infrastructure relays
  - Omitting the delay spread in the relay network
    - In practice, the relayed signal arrives at the destination later than the direct signal due to longer propagation path and processing delay at the relay
      - We assume narrowband signal with symbol period considerably longer than the total delay spread or consider a single subcarrier in OFDM
- Offers also some relevant benefits:
  - Separate channels are not required for source and relay transmissions
  - The two signals are inherently combined without using a more complex receiver
  - The relay is transparent to the destination

Instantaneous End-to-End SNR

- We assume no channel phase information is exploited in the transmitters which makes the signals sum up incoherently
- The instantaneous end-to-end SNR can be formulated as

$$\gamma = \frac{\left|\sqrt{E_{\mathrm{RD}}}h_{\mathrm{RD}}\beta\sqrt{E_{\mathrm{SR}}}h_{\mathrm{SR}} + \sqrt{E_{\mathrm{SD}}}h_{\mathrm{SD}}\right|^2 P_{\mathrm{S}}}{\left|\sqrt{E_{\mathrm{RD}}}h_{\mathrm{RD}}\beta\right|^2 \sigma_{\mathrm{R}}^2 + \sigma_{\mathrm{D}}^2}$$

$$\mathcal{E}_{\psi}\left[\gamma\right] = \frac{\gamma_{\mathrm{SR}}\gamma_{\mathrm{RD}} + (\bar{\gamma}_{\mathrm{SR}} + 1)\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SR}} + \gamma_{\mathrm{RD}} + 1},$$

- The instantaneous link SNRs are γ_{SR}, γ_{RD}, γ_{SD} and the corresponding average SNRs are denoted with overbars
- Average Tx power in the source: P_{S}

  Average Tx power in the relay: P_{R}

Average End-to-End SNR

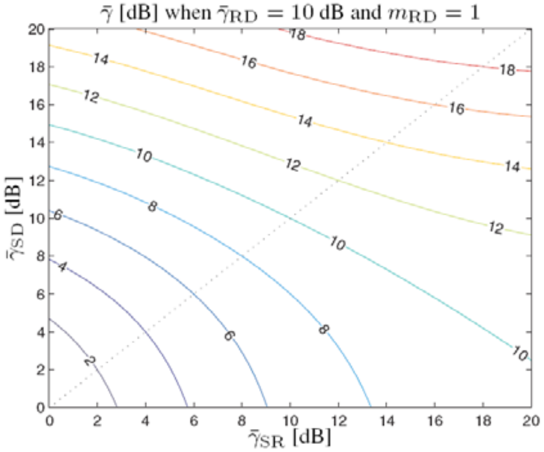
- Let us solve $\bar{\gamma} = \mathcal{E}\left[\dfrac{\bar{\gamma}_{\mathrm{SR}}\gamma_{\mathrm{RD}} + (\bar{\gamma}_{\mathrm{SR}}+1)\bar{\gamma}_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SR}} + \gamma_{\mathrm{RD}} + 1}\right]$
- In a single-frequency relay network:

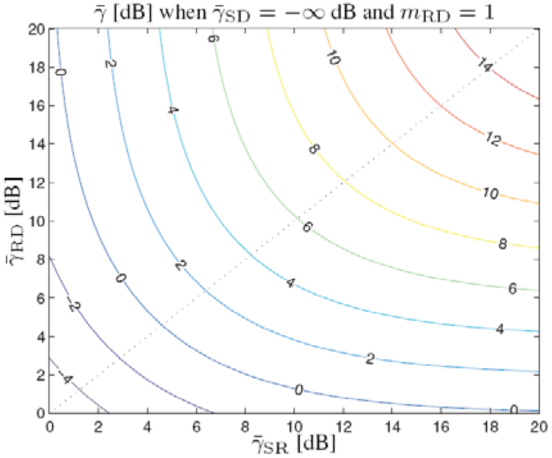
  $$\bar{\gamma} = \bar{\gamma}_{\mathrm{SD}} + (\bar{\gamma}_{\mathrm{SR}} - \bar{\gamma}_{\mathrm{SD}}) m_{\mathrm{RD}} e^{A} E_{m_{\mathrm{RD}}+1}(A)$$

  - where $A = m_{\mathrm{RD}} \frac{\bar{\gamma}_{\mathrm{SR}}+1}{\bar{\gamma}_{\mathrm{RD}}}$
  - If SD channel is better than SR channel then relay employment decreases the SNR compared to mere direct link transmission due to noise amplification
- Without the direct link:

  $$\bar{\gamma} = \bar{\gamma}_{\mathrm{SR}} m_{\mathrm{RD}} e^{A} E_{m_{\mathrm{RD}}+1}(A)$$

  - Both channels have to be roughly equally good, or otherwise receiver or relayed noise limits the end-to-end SNR





Upper Bound of the Average End-to-End SNR

- The exact SNR formula seems to be too intractable for calculating power allocation in closed form
- A property of the exponential integral is used: $e^{A}E_{n}(A) \leq \frac{1}{A+n-1}$
    - Thus, in a single-frequency relay network:

$$\bar{\gamma} \leq \bar{\gamma}^{\mathrm{UB}} = \frac{\bar{\gamma}_{\mathrm{SR}}\bar{\gamma}_{\mathrm{RD}} + (\bar{\gamma}_{\mathrm{SR}}+1)\bar{\gamma}_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SR}}+\bar{\gamma}_{\mathrm{RD}}+1}$$

    - And without the direct link:

$$\bar{\gamma} \leq \bar{\gamma}^{\mathrm{UB}} = \frac{\bar{\gamma}_{\mathrm{SR}}\bar{\gamma}_{\mathrm{RD}}}{\bar{\gamma}_{\mathrm{SR}}+\bar{\gamma}_{\mathrm{RD}}+1}$$

- The upper bounds tighten when the Nakagami fading parameter m_{RD} increases
    - Knowledge on m_{RD} is not required in the upper bound!
- The upper bounds give the exact average SNRs, if the relay-destination channel is non-fading

Illustration of the Power Allocation

- We want to determine power allocation *p* that maximizes the average end-to-end SNR
  - $P_R = pP_S$, $P_S + P_R = 1$
- Illustration: the upper bounds are not tight, but SNRs are quite flat near their maxima
  - Thus, SNR loss is not very large, if we use sub-optimal power allocation factor p^* obtained by maximizing the upper bound

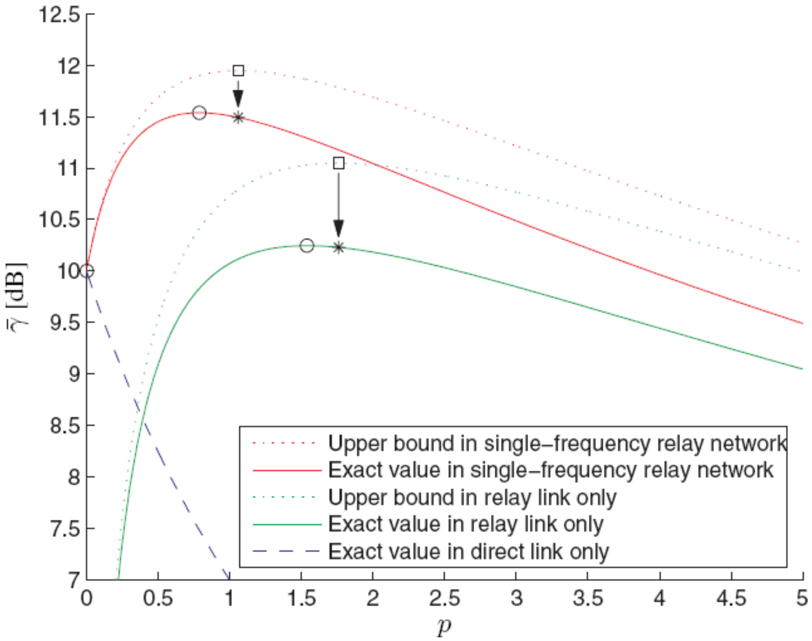


Figure 4: The average end-to-end SNR in the single-frequency fixed-gain relay network with varying power allocation. The maxima of the upper bounds are denoted with (□). The optimal power allocation p^{opt} and the sub-optimal power allocation p^* are shown by (○) and (∗), respectively.

Formulation of the Power Allocation Problem

- We want to determine power allocation *p* that maximizes the average end-to-end SNR
    - Sum power constraint: $P_R = pP_S$, $P_S + P_R = 1$
    - Denote $a_{SR} = E\{\gamma_{SR}\}/P_S$, $a_{RD} = E\{\gamma_{RD}\}/P_R$, and $a_{SD} = E\{\gamma_{SD}\}/P_S$
- We solve the exact optimization problem using numerical tools:

$$p^{\mathrm{opt}} = \arg\max_{p\geq 0}\left\{\frac{a_{\mathrm{SD}} + (a_{\mathrm{SR}} - a_{\mathrm{SD}})m_{\mathrm{RD}}e^{A}E_{m_{\mathrm{RD}}+1}(A)}{p+1}\right\}$$

- Optimization of upper bound can be solved in closed form:

$$p^{*} = \arg\max_{p\geq 0}\left\{\frac{p\,a_{\mathrm{SR}}a_{\mathrm{RD}} + (a_{\mathrm{SR}} + p + 1)a_{\mathrm{SD}}}{(p+1)a_{\mathrm{SR}} + p(p+1)a_{\mathrm{RD}} + (p+1)^{2}}\right\}$$

- It will be shown that performance of the sub-optimal power allocation using the upper bound is very close to the performance of the optimal power allocation

Sub-Optimal Closed-Form Solution

- The fractional function to be maximized is concave and its maximum is found by solving for the roots of its derivative
- Thus, in a single-frequency relay network:

$$p^* = \left(\frac{\sqrt{a_{\mathrm{SR}} a_{\mathrm{RD}} (a_{\mathrm{SR}} - a_{\mathrm{SD}})(a_{\mathrm{RD}} - a_{\mathrm{SD}})}}{a_{\mathrm{SR}} a_{\mathrm{RD}} + a_{\mathrm{SD}}} - \frac{\sqrt{(a_{\mathrm{SR}} + 1)(a_{\mathrm{RD}} + 1) a_{\mathrm{SD}}}}{a_{\mathrm{SR}} a_{\mathrm{RD}} + a_{\mathrm{SD}}} \right) \sqrt{\frac{a_{\mathrm{SR}} + 1}{a_{\mathrm{RD}} + 1}}$$

  - If $a_{\mathrm{SR}} < a_{\mathrm{SD}}$ or $a_{\mathrm{RD}} < a_{\mathrm{SD}}$ then $p^* = 0$
- Without the direct link the solution reduces to

$$p^* = \sqrt{\frac{a_{\mathrm{SR}} + 1}{a_{\mathrm{RD}} + 1}} = \sqrt{\frac{\frac{E_{\mathrm{SR}}}{\sigma_{\mathrm{R}}^2} + 1}{\frac{E_{\mathrm{RD}}}{\sigma_{\mathrm{D}}^2} + 1}}$$

Evaluation of the Sub-Optimal Power Allocation

- The relay is located on a segment of a line whose end points are the source and the destination
  - The span of the direct link between the source and the destination is normalized to 1
  - The normalized distance between the source and the relay is denoted by *d*
  - E_{SD} = 10 dB, and noise powers are 1
  - An exponential path loss model with exponent 3
- The difference between the optimal power allocation and the sub-optimal power allocation using the upper bound is barely distinguishable

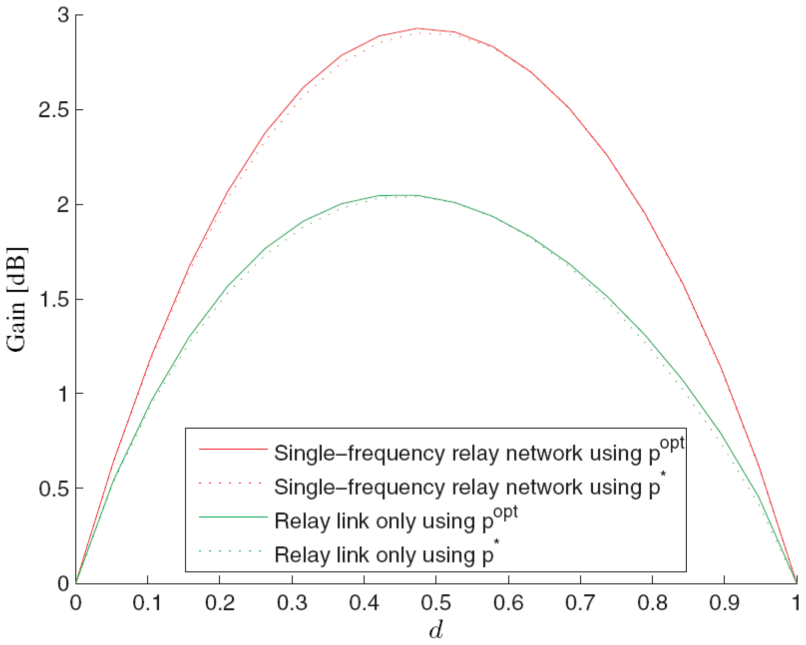


Figure 5: The relaying SNR gain over mere direct link transmission.

Conclusion

- We consider a simple and cheap relay
  - Due to single-frequency implementation it is transparent to the user
  - Fixed-Gain operation requires only average channel state information
- We derive instantaneous and average end-to-end signal to noise ratios (SNRs) for the single-frequency fixed-gain relay network
- For transmit power allocation between the source and the relay, performance measure is the average end-to-end SNR when the total transmit power in the network is kept constant
  - This performance criterion can be used to maximize the relaying SNR gain, or to optimize total power consumption in the network
  - We define the exact power allocation problem and solve it numerically
  - Closed-form expressions for sub-optimal power allocation are derived by maximizing an upper bound of the average end-to-end SNR
    - This sub-optimal power allocation is shown to perform very close to the optimal power allocation and it requires knowledge of only the average SNRs
- Finally, relay deployment is demonstrated to offer SNR gain compared to mere direct link transmission

Thank you!

Questions?

Comments?