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Diversity Analysis of a Parallel Amplify and Forward Relay Network

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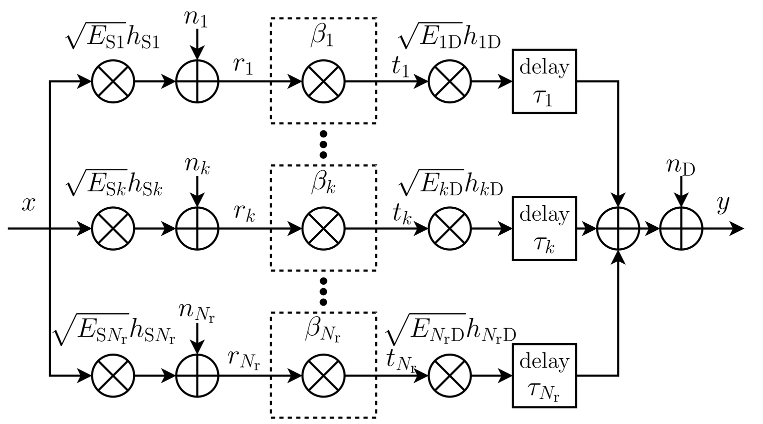
Introduction

- The $(1 \times N_r \times 1)$ relay network model: The source broadcasts, the relays amplify-and-forward (AF) and the destination receives a superposition:



$$r_k(\tau) = \sqrt{E_{Sk}} h_{Sk} x(\tau - \tau_{Sk}) + n_k$$
$$t_k(\tau) = \beta_k r_k(\tau)$$
$$y(\tau) = \sum_{k=1}^{N_r} \sqrt{E_{kD}} h_{kD} t_k(\tau - \tau_{kD}) + n_D$$
$$\tau_k = \tau_{Sk} + \tau_{kD}$$

  – In practical wireless networks spatial separation causes different propagation delays for the signal components and the destination observes a multipath channel.

- The received energy E_{rcv} at the destination is used to quantify diversity with an ideal wideband receiver. We define a diversity metric given by

$$\Delta = \frac{\mathrm{Var}\left[E_{rcv}\right]}{(\mathcal{E}\left[E_{rcv}\right])^2},$$

  which characterizes the severity of fading that offers one but not the only method to analyze performance.

- The diversity metric Δ is derived as a function of signal bandwidth W and the average received SNR at the relays.

Single Relay Link

- The diversity metric of a single relay channel is simply

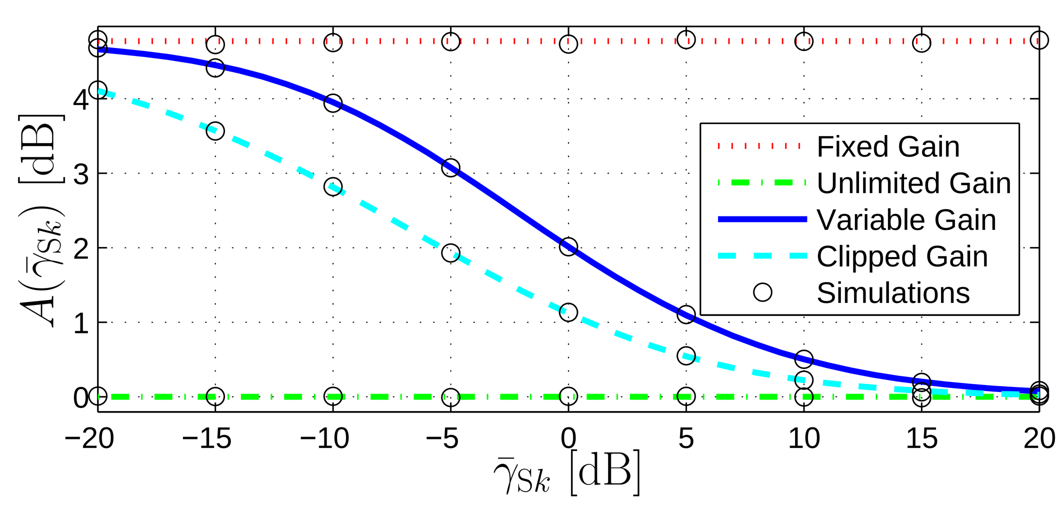
$$A(\bar{\gamma}_{Sk}) = \frac{\mathrm{Var}\left[|\alpha_k|^2\right]}{(\mathcal{E}\left[|\alpha_k|^2\right])^2}, \text{ where } \alpha_k = \sqrt{E_{Sk}} h_{Sk} \beta_k \sqrt{E_{kD}} h_{kD}.$$

- For the four AF protocols with Rayleigh channels:

	Fixed Gain (FG)	Unlimited Gain (UG)	Variable Gain (VG)	Clipped Gain (CG)
β_k	$\frac{1}{\sqrt{E_{Sk}+\sigma_k^2}}$	$\frac{1}{\sqrt{E_{Sk}+\sigma_k^2}\lvert h_{Sk}\rvert}$	$\frac{1}{\sqrt{E_{Sk}\lvert h_{Sk}\rvert^2+\sigma_k^2}}$	$\begin{cases} \frac{\sqrt{G}}{\sqrt{E_{Sk}+\sigma_k^2}\lvert h_{Sk}\rvert}, & \lvert h_{Sk}\rvert > \frac{\sqrt{G}}{\sqrt{E_{Sk}+\sigma_k^2}\beta_{max}^{CG}} \\ \beta_{max}^{CG}, & \text{otherwise} \end{cases}$
$A(\bar{\gamma}_{Sk})$	3	1	$4 \cdot e^{-\frac{1}{\bar{\gamma}_{Sk}}} \frac{E_2(\frac{1}{\bar{\gamma}_{Sk}}) - E_3(\frac{1}{\bar{\gamma}_{Sk}})}{E_2^2(\frac{1}{\bar{\gamma}_{Sk}})} - 1$	$4 \cdot \frac{1-(1+\frac{G}{\bar{\gamma}_{Sk}+1})e^{-\frac{G}{\bar{\gamma}_{Sk}+1}}}{\left(1-e^{-\frac{G}{\bar{\gamma}_{Sk}+1}}\right)^2} - 1$

  – The average received SNR in the kth relay is $\bar{\gamma}_{Sk} = \frac{E_{Sk}}{\sigma_k^2}$.

- Illustration:



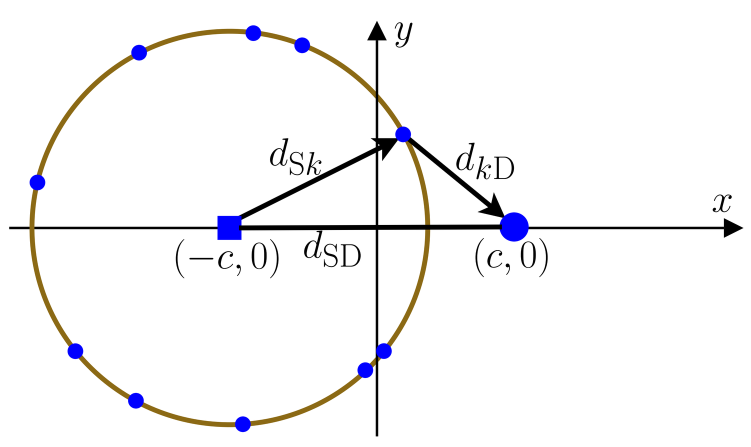
  – To achieve $A(\bar{\gamma}_{Sk}) < 1$, the relays would have to be able to also invert the forward (relay-destination) channels, i.e., feedback information is required.
  – Only if α_k has a constant amplitude and uniformly distributed random phase then $A(\bar{\gamma}_{Sk}) = 0$. This would correspond to a virtually line-of-sight channel.
  – In practice, AF relays would have to operate on relatively high input SNR to provide a reasonable performance. Thus, the performance of the practical VG and CG protocols is very close to the theoretical UG prototol. The CG protocol proposed in this paper is slightly better than Variable Gain protocol.

Parallel Relay Network

- Let us assume that the cascade of transmit and receive filters is an ideal low-pass filter with a cut-off frequency $W/2$, and the constant η is selected in such a way that $\mathcal{E}\left[E_{rcv}\right] = 1$. Thus,

$$\frac{\mathrm{Var}\left[E_{rcv}\right]}{\eta^4} = \sum_{k=1}^{N_r} \mathrm{Var}\left[|\alpha_k|^2\right] + \sum_{k_1=1}^{N_r} \sum_{k_2 \neq k_1} \mathcal{E}\left[|\alpha_{k_1}|^2\right] \mathcal{E}\left[|\alpha_{k_2}|^2\right] \mathrm{sinc}^2\left(W(\tau_{k_1} - \tau_{k_2})\right).$$

- An example configuration that may arise in downlink when relays are used to extend the coverage of a cellular base station:

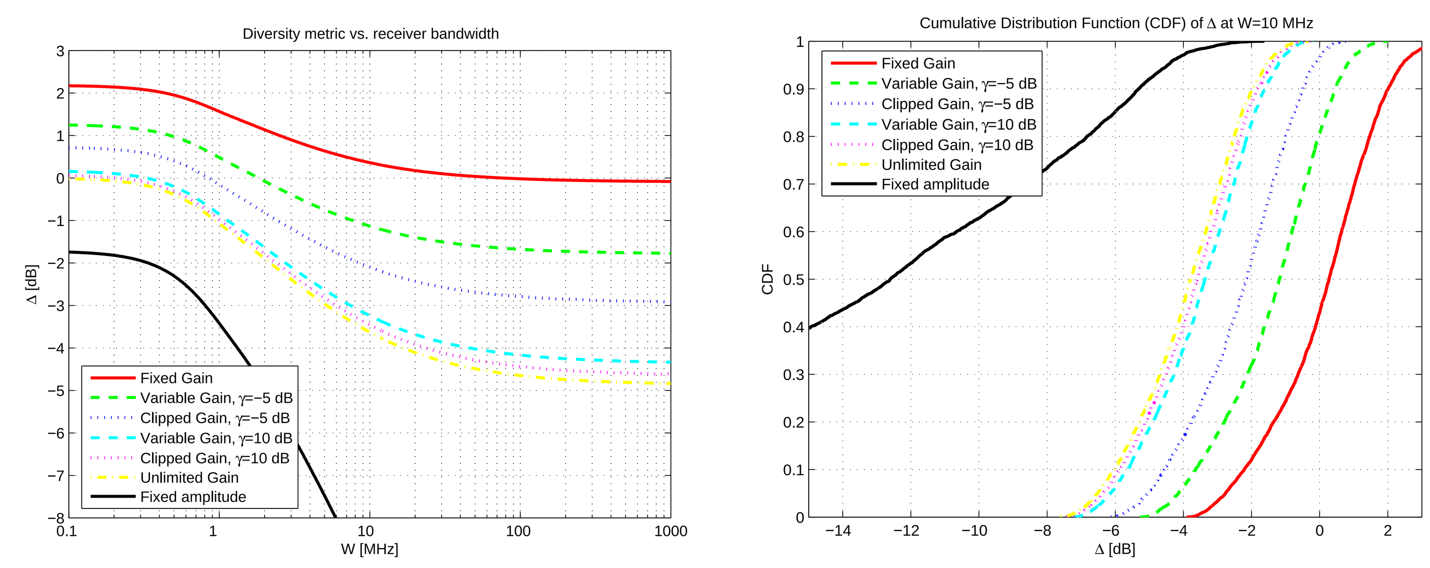


  – Assumptions:
    * An exponential path loss model with exponents μ_{Sk} and μ_{kD} is used for the source-relay and relay destination links, respectively.
    * The transmit powers are the same in every relay.
    * Noise levels are equal in all relays.

  – Now $\bar{\gamma}_{Sk} = \bar{\gamma}$ is the same for all relays, which results in a nice expression for the diversity metric:

$$\Delta = A(\bar{\gamma})\eta^4 \sum_{k=1}^{N_r} E_{kD}^2 + \eta^4 \sum_{k_1=1}^{N_r} \sum_{k_2 \neq k_1} E_{k_1 D} E_{k_2 D} \mathrm{sinc}^2\left(W(\tau_{k_1} - \tau_{k_2})\right).$$

  – In high bandwidth scenario ($W \to \infty$), the latter term goes to zero and Δ is directly proportional to $A(\bar{\gamma})$.
  – The minimum of $\eta^4 \sum_{k=1}^{N_r} E_{kD}^2$ being $\frac{1}{N_r}$ is obtained only if E_{kD} are equal for all k. Thus, $\Delta \geq \frac{A(\bar{\gamma})}{N_r}$, which is reached at high bandwidth with equal E_{kD} for all k.
  – Simulation results:



Diversity metric simulated over 10^4 instances of the circular relay network.

Conclusion

- An analytical diversity metric is defined for spatially distributed parallel relay networks. Closed-form expressions of the metric are derived for the most important amplify-and-forward protocols.
  – We propose a new relaying protocol by clipping the relay gain that performs better in terms of the diversity metric.
  – An ideal channel-inverting protocol can be used as a benchmark for practical relaying protocols at least with relatively high relay input SNR.
  – Furthermore, we notice that the diversity metric in practical relay networks may not be close to the optimal value $1/N_r$ of N_r relays, because signals arriving from different relays have high power differences due to spatial separation.

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