Delay Spread and Its Effect on Bandwidth in Gaussian Parallel Relay Networks

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Outline

- Introduction
- System model
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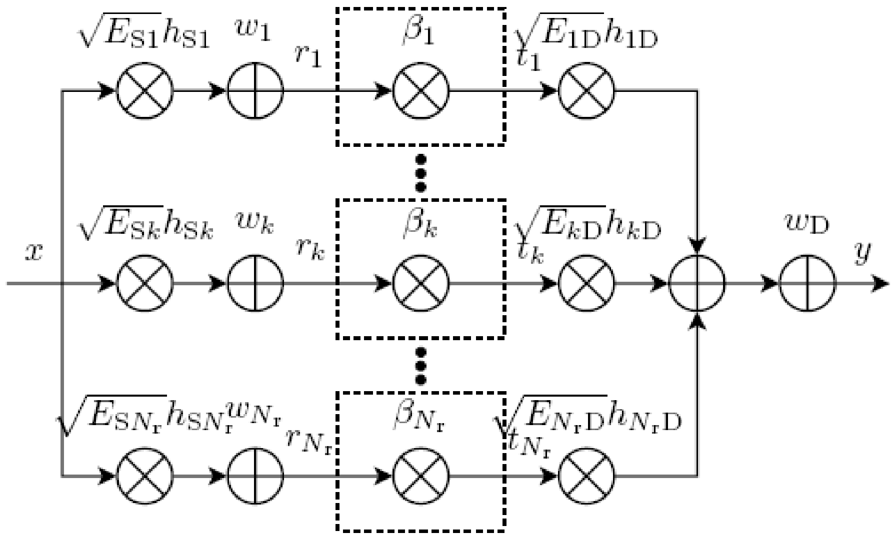
Introduction

- The Gaussian parallel relay network [1]
- Performance scaling analysis of wireless ad-hoc networks [2-6]
- Spatial delay spreading is often neglected
  - Few articles have proposed solutions to cope with the delay spread in parallel relay networks, e.g. [7-8]
  - Omitting delays of relayed signals is not feasible in practical cases (spatially separated network nodes)
    - In rural areas, the reference architecture for DVB-H may contain a single main transmitter and 10-20 amplify-and-forward relays [9]
    - OFDM: It is important that the delay spread of the received sum signal remains within the limits defined by the cyclic prefix
- The purpose of this paper is to study the relationship between signal delays, bandwidth and diversity
  - Narrowband vs. wideband reception

System Model

- Parallel amplify-and-forward relay network, delays omitted in the figure



- Half-duplex frequency division in the relays

$$r_k(\tau) = \sqrt{E_{\mathrm{S}k}}h_{\mathrm{S}k}x(\tau - \tau_{\mathrm{S}k}) + w_k$$

$$t_k(\tau) = \beta_k r_k(\tau), \quad k = 1, 2, \ldots, N_\mathrm{r}$$

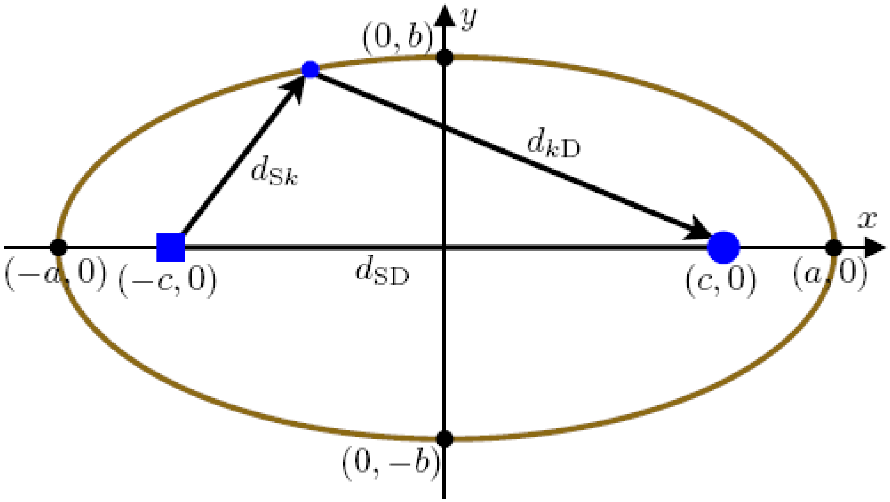
$$y(\tau) = \sum_{k=1}^{N_\mathrm{r}} \sqrt{E_{k\mathrm{D}}}h_{k\mathrm{D}}t_k(\tau - \tau_{k\mathrm{D}}) + w_\mathrm{D}$$

Relay Operation Modes

- Three operation modes depending on the level of available channel state information (CSI):
  - 1) *No CSI*: If the relays do not use any channel state information, they forward with a fixed gain ($\beta_k = 1$) and the effective channels between the source and the destination become double Rayleigh.
  - 2) *Receive CSI*: If the relays know the instantaneous amplitude of the backward channel, they can equalize its fluctuation, i.e., $|h_{\mathrm{S}k}\beta_k| = 1$. The effective channels become (single) Rayleigh.
  - 3) *Receive and transmit CSI*: If the relays know both backward and forward channels, the effective channels can be turned into fixed amplitude channels, i.e., $|h_{\mathrm{S}k}\beta_k h_{k\mathrm{D}}| = 1$.

Narrowband Reception (1)

- Relays that produce the same delay are located at an ellipse



- Assume that the relays are uniformly distributed

$$N_{\mathrm{r}} \leq N(\tau_{\max}) = \rho\pi\sqrt{\left(\frac{v\tau_{\max}}{2}\right)^4 - \left(\frac{v\tau_{\max}}{2}\right)^2 c^2}$$

- Thus, the maximum delay $\tau_{\max} \geq \frac{1}{v}\sqrt{2c^2 + 2\sqrt{c^4 + 4\left(\frac{N_{\mathrm{r}}}{\rho\pi}\right)^2}}$

Narrowband Reception (2)

- The maximum available bandwidth:
  - The maximum delay spread must be less than a fraction *δ* of the symbol duration, i.e.,

$$\tau_{\max} - \tau_{\min} \leq \delta T_{\mathrm{s}}$$

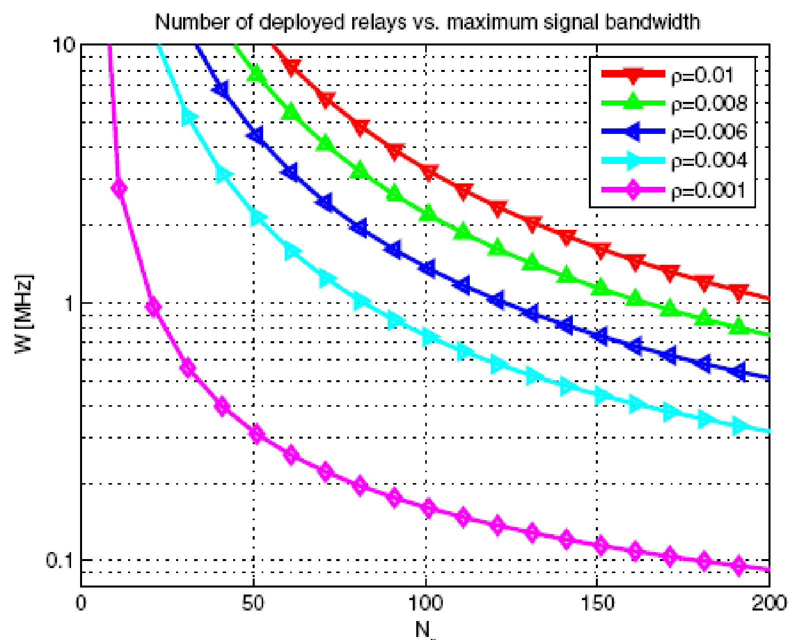
  - Thus, bandwidth is limited by

$$W \leq \frac{\delta v}{\sqrt{2c^2 + 2\sqrt{c^4 + 4\left(\frac{N_{\mathrm{r}}}{\rho\pi}\right)^2}} - 2c}$$

- Asymptotically the available bandwidth scales as

$$W = \mathcal{O}\left(\frac{1}{\sqrt{N_{\mathrm{r}}}}\right)$$

- For example with *δ*=0.1, maximum available bandwidths for different relay densities *ρ*:



Wideband Reception (1)

- A diversity metric [10] for the received energy E_{rcv}

$$\Delta = \frac{\text{Var}\left[E_{\text{rcv}}\right]}{\left(\mathcal{E}\left[E_{\text{rcv}}\right]\right)^2}$$

- Applying results from [14] to a relay network, we get an analytical expression for the diversity metric

$$\Delta = \sum_{k_1=1}^{N_{\text{r}}} \sum_{k_2 \neq k_1} G_{k_1} G_{k_2} \text{sinc}^2[W(\tau_{k_1} - \tau_{k_2})] + \alpha$$

where $\alpha = A \sum_{k=1}^{N_{\text{r}}} G_k^2$ and $G_k = E_{\text{S}k} E_{k\text{D}}$

- The constant values are $A = 0$, $A = 1$ and $A = 3$ for fixed amplitude (Both CSI), Rayleigh (Receive CSI), and double-Rayleigh (No CSI) channels, respectively.

Wideband Reception (2)

- Upper bound of diversity metric

$$\Delta \leq \beta / W^2 + \alpha$$

where

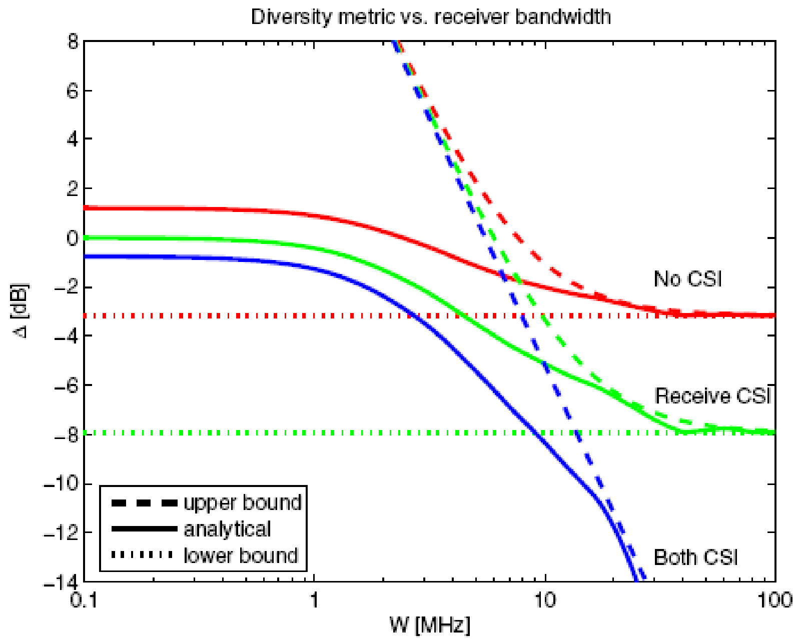
$$\beta = \frac{1}{\pi^2} \sum_{k_1=1}^{N_r} \sum_{k_2 \neq k_1} \frac{G_{k_1} G_{k_2}}{(\tau_{k_1} - \tau_{k_2})^2}$$

- Lower bound of diversity metric

$$\Delta \geq \alpha$$

- Diversity converges to a floor value after a “knee” bandwidth

- Diversity behaviour of a random example network
  - c=100m, ρ=0.0001 1/m^2, τ_{max}=1.3 μs, N_r=12 relays



Conclusion

- Gaussian parallel relay networks are narrowband in nature.
- Narrowband reception
  - Available bandwidth is asymptotically upperbounded by $\mathcal{O}(\frac{1}{\sqrt{N_\mathrm{r}}})$
  - Maximum available bandwidth is reached, when the N_r uniformly distributed relays are located inside an ellipse.
- Wideband reception
  - Diversity improves with increasing bandwidth up to a “knee bandwidth” and then converges to a floor value
  - Only if the relays were able to exploit full channel state information and invert end-to-end channels perfectly, the metric would be arbitrarily small.
- We see future research in developing relaying protocols that take into account delay spread, transceiver dynamics, SNR and power constraints.

Thank you!

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